VOL. 6. No. 9, p. 253-284 SEPTEMBER 1941

Philips Technical Review

DEALING WITH TECHNICAL PROBLEMS
RELATING TO THE PRODUCTS, PROCESSES AND INVESTIGATIONS OF
N.V. PHILIPS' GLOEILAMPENFABRIEKEN

EDITED BY THE RESEARCH LABORATORY OF N.V. PHILIPS' GLOEILAMPENFABRIEKEN, EINDHOVEN, HOLLAND

SEVERAL TECHNICAL PROBLEMS IN THE DEVELOPMENT OF A NEW SERIES OF TRANSMITTER VALVES

by E. G. DORGELO.

621.396.615

The use of shorter and shorter radio waves involves a steady decrease in the dimensions of the transmitter valves. When it is at the same time desirable not to decrease the energy dissipation in the valve, various parts of the valve become relatively more heavily loaded. An indication is given in this article of how several technical problems connected with this are solved in the case of a new series of Philips transmitter valves.

Introduction

Under the influence of the rising standard of performance required in the field of television, a great demand has arisen in recent years for transmitter valves which can work with high efficiency on the wave lengths of 5 to 6 m used in television. The existing types could to some extent be used for such wave lengths, but usually worked at low efficiency, and, moreover, they often involved great difficulties in the design and adjustment of the transmitter.

In connection with this an entirely new series of transmitter valves has been developed, which is suitable not only for long waves but also for the short waves mentioned above. In appearance this type of valve differs from the older types mainly in its much smaller size. This small size was necessary on the one hand to diminish the transit time effects, and on the other hand to make possible a better adaptation to the transmitter by smaller electrode capacities and smaller self- and mutual inductions in the connections.

The series of valves now developed consists of four triodes of about 250, 600, 1 200 and 2 500 W telegraphy output ¹) respectively, three pentodes of about 200, 500 and 1 000 W and a push-pull pentode also of about 1 000 W (see *fig. 1*).

The application of the push-pull principle was justified by the consideration that otherwise at the wave lengths of a few metres many of the advantages of the pentode connection are lost (for a detailed discussion of the advantages and disadvantages of triodes and pentodes we refer to an article published earlier ²). While at longer wave lengths screen grid and suppressor grids function as shielding cage due to their constant potential, and thereby make special decoupling measures (neutrodyning) unnecessary, at very short waves this is no longer true. The self-induction of the connections forms such a high impedance at these frequencies that the screen-grid alternating current causes the potential of the screen grid to vary appreciably.

A solution of this is to connect two valves in push-pull connection and supply the corresponding electrodes together through a single line, so that the alternating currents cancel each other in this line. There then occurs no A.C. voltage along these connections. Between the common connection and each of the electrodes, however, there remain connections which are not in common. These connections may also still have too much impedance at very short waves, so that recourse must then be had to reducing their impedance with the help of series resonance. The filament connection also must often be tuned in this way (see *fig. 2*).

These precautions are, however, at least with wave lengths of a few metres, not yet necessary when the two electrode systems are assembled in a single bulb

---

¹) By "telegraphy output" we mean here the maximum output in class C adjustment. With this arrangement the control grid voltage is so strongly negative that anode current flows for less than half a period.

²) J. P. Heyboer, Philips techn. Rev., 2, 257, 1937.

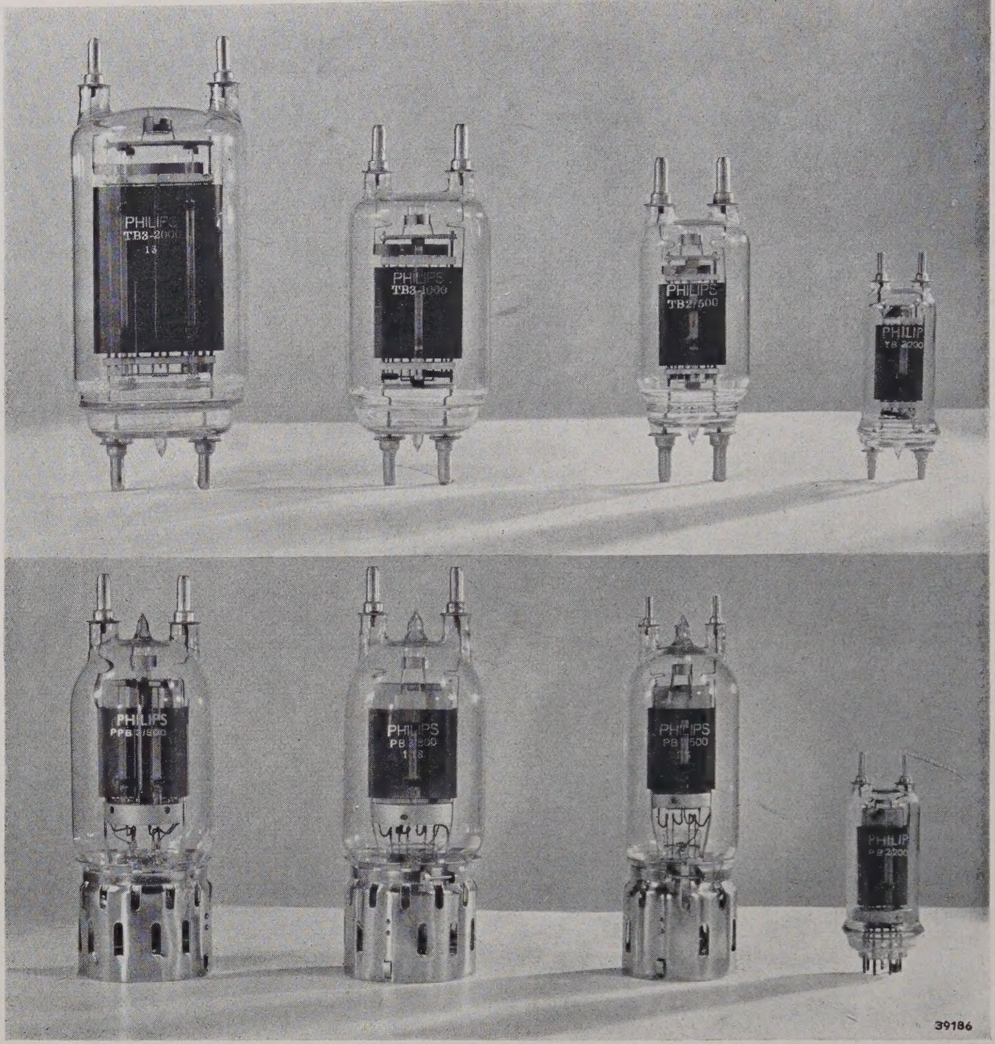


Fig. 1. Photograph of a series of new transmitter triodes and pentodes which were developed on the principles set forth in this article. The output in telegraphy adjustment, class C, for the series of pentodes (lower row) is, from left to right: 1 000, 1 000, 500, 200 W, and for the series of triodes (upper row) it is 2 500, 1 200, 600, 250 W.

and joined with the shortest possible connections. A particularly compact solution is obtained by surrounding the two cathodes and control grids with a single screen grid and suppressor grid (*fig. 3*). This concept is realized in the push-pull pentode PPB 3/800 which can work entirely without extra tuning to the shortest wave lengths which it can reach (2.5 metres).

For the fundamental requirements which every transmitter valve must satisfy we may refer to an earlier article in this periodical [3]). In addition to the points touched upon here, which are connected with form and manner of construction of the modern transmitter valves, the requirement of small size also raises technical problems which will be discussed in the following sections.

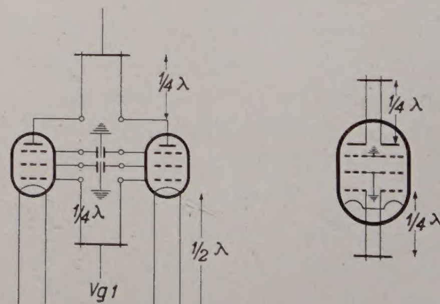


Fig. 2. *a*) Diagram of two pentodes in push-pull connection. The parts of suppressor and screen grid connection not possessed in common are tuned with the help of variable series capacities. The regulation of the filament impedance is by means of Lecher systems of the length $^1/_2\ \lambda$. As oscillation circuits in anode and control grid circuits Lecher systems are also used (length $^1/_4\ \lambda$).

b) In the push-pull pentode PPB 3/800 only the grid and anode circuit are tuned. The filaments are connected with very short connections inside the valve. Furthermore there is only one common suppressor and screen grid.

---

[3]) H. G. Boumeester, Philips techn. Rev., 2, 115, 1937.

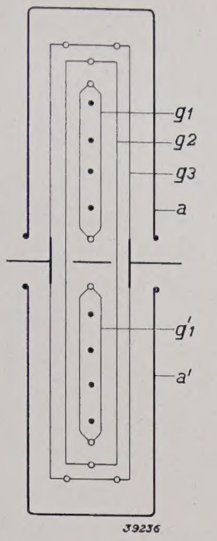


Fig. 3. Cross section of the electrode system of the push-pull pentode PPB 3/800. The shielding between the two halves of the anode prevents electrons from the right-hand cathode from reaching the left-hand anode, and *vice versa.*

Fundamental differences between the modern and the older types of valves

We shall here first mention those parts of a valve to which special attention must be paid, and later discuss two of them in more detail.

a) Grids and anode

Small dimensions mean small surfaces, and thus with a given power a high specific dissipation. It is therefore necessary to use for the electrode system a material which can withstand high temperatures. Nickel and iron are less suitable because of their fairly great volatility; molybdenum, tungsten and tantalum are better. In the new series of Philips valves molybdenum has been used exclusively for anode and grids, and all connections are clinched with molybdenum rivets or welded directly.

b) Cathode

The high working temperature, which in the case of the anode may amount to 800—900 °C, prevents the use of anything but pure or thoriated tungsten for the cathode. Oxide cathodes would be too much overloaded due to heating from the other electrodes. Considering the high emission of thoriated tungsten compared with pure tungsten (about 70 mA/W *versus* 6 mA/W) the former has been chosen. With a relatively low filament power a good anode current can now be obtained, so that these valves can deliver a satisfactory output with a fairly low anode voltage (3 000 V or lower). Especially on the shortest waves where the circuit losses become very great at high voltage this is important (see *fig. 4*).

c) Bulb

The fact that great power is dealt with in a small electrode system necessitates making the covering of the valve, the bulb, either very large in diameter or of a kind of glass which does bot easily soften. When hard glass is used the bulb can be made quite narrow, so that short leads with slight self- and mutual induction are obtained. The use of this glass at first presented difficulties because of the fact that the existing kinds of hard glass became slightly conducting, especially at the high working temperature here prevailing, so that particularly the fused-in leads were electrolytically attacked. By making use of newly developed glasses and a fusing-in technique borrowed from the quartz lamps, however, it was possible entirely to overcome these difficulties, as will be described in the following.

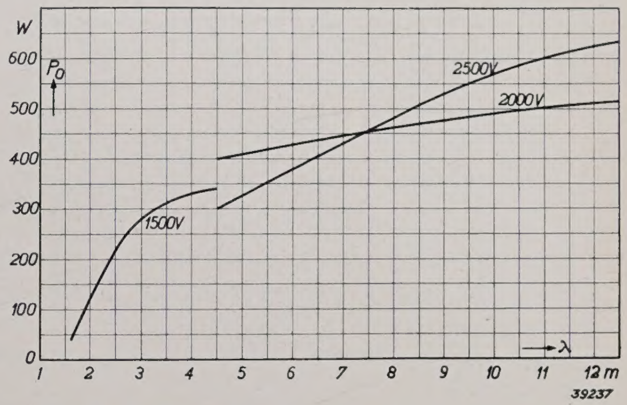


Fig. 4. Output of a PPB 3/800 valve as a function of the wave length. It may be seen that it is more advantageous to lower the anode voltage to a definite limit at short wave lengths, since the losses in the circuits of the transmitter increase too rapidly with the voltage. In connection with this effect it is desirable to construct valves for short wave lengths for a relatively low anode voltage.

d) Getter

Another result of the high temperature was the difficulty of maintaining a sufficiently high vacuum during use. Ordinary getters such as barium and magnesium could not be used here for various reasons. Aside from the great hindrance to the heat radiation formed by the mirror deposited on the bulb wall, the mirror also constitutes an undesired and badly reproducible capacity with respect to the electrodes. At a bulb temperature of 300-350 °C the vapour pressure of barium and magnesium is also already so high that one may scarcely speak of a "vacuum".

Zirconium, whose getter properties have already been discussed in this periodical [4]), lacks the disadvantages mentioned. It can easily be deposited on the electrodes in powder form and thereby automatically takes on the temperature necessary for satisfactory functioning.

---

[4]) J. D. Fast, Philips techn. Rev. 5, 217, 1940.

In the following we shall go into somewhat more detail about the bulb and the getter.

The bulb and the electrode leads through the glass

As already mentioned, electrical conduction easily occurs in the glass of the bulb, especially between the fused-in leads. Conduction in glass, like the electrolysis of liquids, is based upon the movement of ions. In the conduction in glass substances are also deposited at the poles. The presence of these substances results in general in a lessened reliability of the fused-in leads. The oxide layer usually present on the metal is discoloured, gas bubbles are formed in the boundary layer and after some time the valve begins to leak. As a result of di-electric losses which always accompany the electrical conduction in the case of bulbs of transmitter valves, this phenomenon becomes more disturbing the shorter the wave length.

Since all the ions which occur in glass are not equally mobile, by a suitable choice of composition of the glass a considerable improvement of the insulating power can be attained. During recent years types of glass have been successfully manufactured whose electrical conductivity is many thousand times smaller than that of ordinary hard glasses (*fig.* 5).

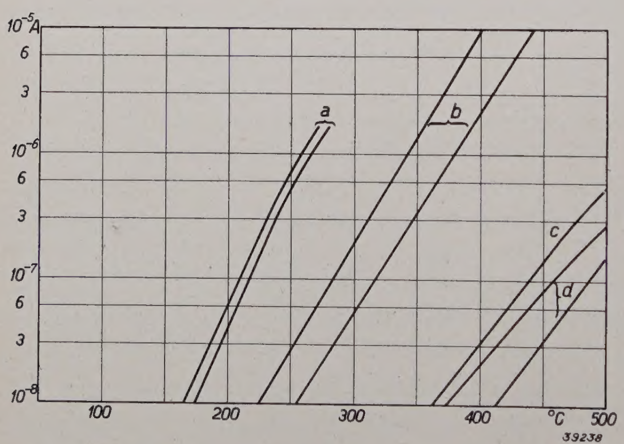


Fig. 5. Electrolytic conductivity of glass as a function of the temperature. In a piece of glass two wires of 1 mm diameter are fused in over a length of 1 cm at a distance apart of 1 cm. Between the wires a D.C. voltage of 100 V is applied. The current increases approximately according to a power of e with the temperature. Four groups of glass can be distinguished:
a) Borosilicate glass. Softening point 525-550 °C. Resistance low.
b) Soft glass (lead glass). Softening point about 450 °C. Resistance fairly high.
c) Hard glass. Softening point 700-1000 °C. Resistance high.
d) "Electrolysis-free" glasses. Softening point 550-600 °C. Resistance very high.

Another great improvement was the introduction of bare metal leads through glass. By means of the correct technique of fusing in, in combination with the use of suitable kinds of glass, it was found possible to fuse in leads of molybdenum and tungsten entirely free of oxide. Such bare leads are distinguished from the earlier leads by the fact that even at working temperatures of 400 °C and more they remain absolutely vacuum tight.

Leads through glass of this type are used in the pentodes of the series here described for the base leads (filament, control, screen and suppressor grids).

The top leads (anode and a second connection of the suppressor grid) have a considerably larger high-frequency current to withstand, since the output circuit is connected to them. Of themselves the bare molybdenum or tungsten leads through glass would be quite satisfactory here; it is, however, difficult to fasten a suitable binding post arrangement to them. Direct connection is out of the question because of the brittleness of fused-in molybdenum or tungsten.

A much stronger arrangement is obtained by the use of heavy copper pins which are fused to the bulb by means of a "fernico" ring soldered to them. These leads can easily withstand 50 A or more.

In the triodes of this series copper with fernico is also used for the base connections (filament).

The zirconium getter

Pure zirconium has a very strong tendency, depending on the temperature, to bind gases. When used in powder form a large active surface is obtained while gas is then taken up even at low temperatures. In general it may be said that for binding oxygen and nitrogen red heat is necessary, while hydrogen is taken up only by the non-glowing parts (see the article referred to in footnote [4]).

In a transmitter valve all these temperatures are as a rule present at the same time. Thus in the case of the largest of the valves here described the maximum anode temperature is about 800 °C, the lowest about 300 °C, while connection strips and the like may be still colder. Since a good vacuum is necessary here in the first place for maintaining the cathode emission (tungsten-thorium cathodes are very sensitive to oxygen), it is obvious that the zirconium powder should be placed in the immediate vicinity of the filament.

In the case of the pentodes of this series the screen grid as well as the anode is covered with zirconium. In the case of the triodes also it was natural to make use of such a shielding cage which in this case could only be formed by the control grid. While indeed the control grid of a triode usually has a lower working temperature than the screen grid of a pentode, the getter action is in this case increased by the fact that the zirconium on the control grid has a low electric potential with respect to the surroundings, so that any positive ions present are drawn to it. It was actually found possible to maintain a good vacuum in a triode also by this means.

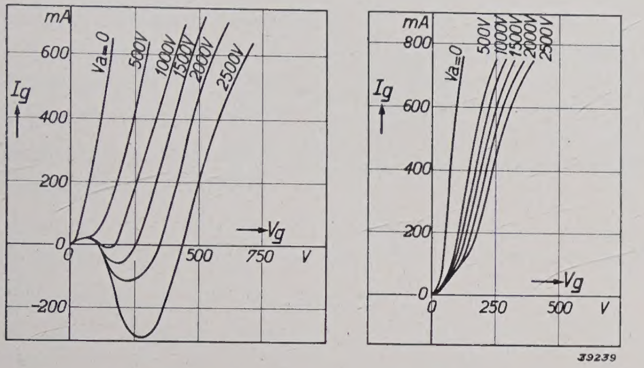


Fig. 6. Grid current characteristics of the valve TB 3/1 000, *a*) without zirconium, *b*) with a very thin layer of zirconium powder on the grid. Because of the fact that the coefficient of secondary emission of molybdenum is greater than unity for electron velocities of several hundred volts, a negative grid current may occur in *a*). In practice, however, a high positive grid voltage is accompanied by a low anode voltage, so that with a well constructed valve the momentary value of the grid current always remains positive.

The covering of electrodes with zirconium has still other results. As was described in the article referred to in footnote [3]), zirconium decreases the secondary emission of the surface upon which it is deposited. Since the total current to an electrode is the sum of the primary emission falling on the electrode and the secondary emission counter to it, influence can be exerted on the total current by covering the electrode with more or less zirconium.

In the case of the control grid the desire is as a rule to make the total current as small as possible. Every increase in the grid current is accompanied by an increase in the excitation power and thus a decrease in the energy amplification. It is therefore best in this case when the secondary emission is as nearly as possible equal to the primary electron current received.

Now in the case of modern transmitter valves this is usually approximately the case even without the deposit of zirconium. They are constructed for low voltage and great current density so that there is so much space charge in the valve that secondary electrons pass through it with difficulty, and the secondary emission current, which of itself is considerably larger than the primary current, is reduced to about the desired value.

Covering the grid with zirconium would in such valves disturb the equilibrium between the currents, and thus have an unfavourable effect. In order to give some idea of the change in grid current caused by covering it with zirconium, several characteristics are shown in *fig. 6*. In the case of the valve with zirconium-covered control grid, used in a high-frequency connection, the grid current was 165 mA and the energy amplification 18 times, compared with 90 mA and 39 times in the case of the ordinary valve. In order to prevent such an increase in the grid current, in the new valves with zirconium getter only those parts of the control grid were covered which are not exposed to a direct bombardment by primary electrons. The parts here involved are the grid rod and in some cases the half of the wire surface which does not face the filament.

In the case of the anode also covering (the inside) of the anode with zirconium affects the characteristic. In *fig. 7* the i_a-V_a characteristics of two 1 000 W transmitter tetrodes are shown. One has a bare molybdenum anode; in the other the anode is covered internally with zirconium. In the latter

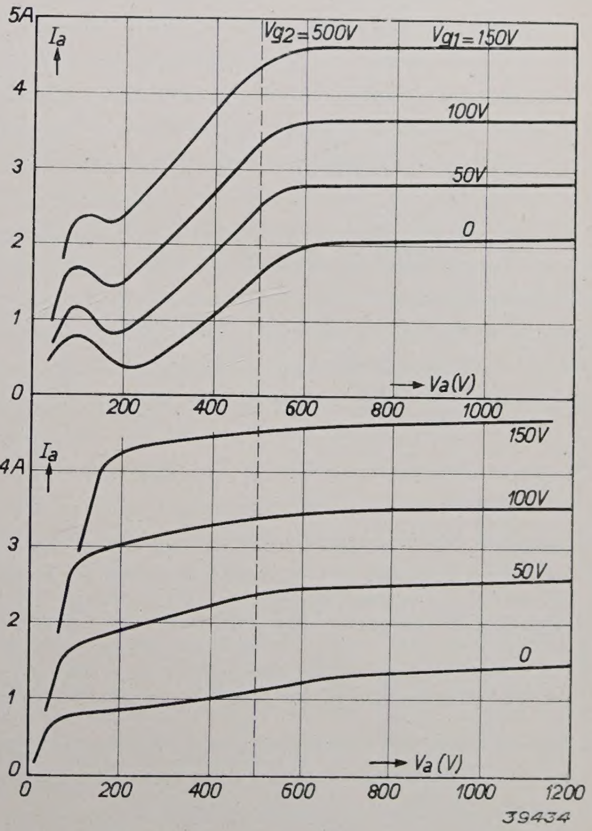


Fig. 7. The i_a-V_a characteristics of two tetrodes. The upper characteristic refers to a tetrode with molybdenum anode; the lower characteristic is obtained when the anode is covered internally with zirconium powder.

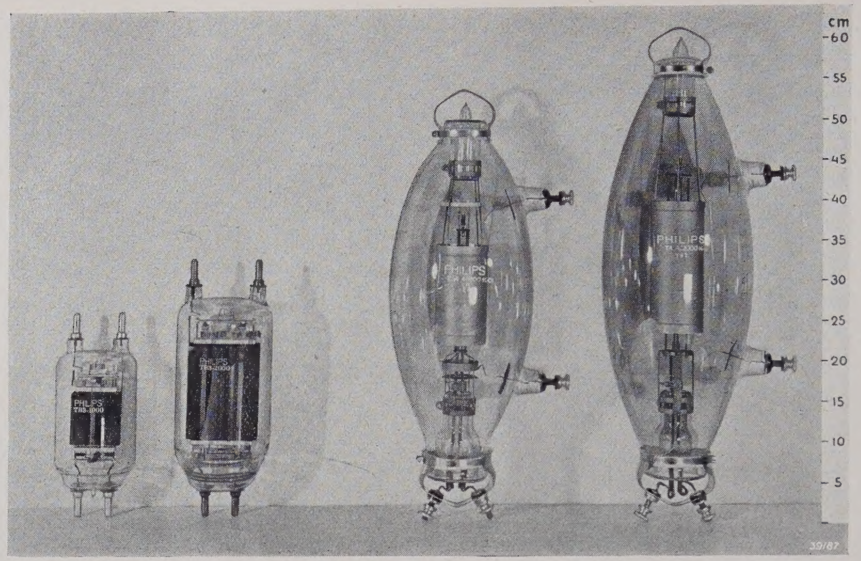


Fig. 8. Two modern transmitter triodes (left) and two used formerly (right) of about the same output. The photograph illustrates the enormous decrease in external dimensions.

case the familiar link which occurs as a result of the secondary emission for $V_a < V_{g2}$ has disappeared almost entirely, and a characteristic is obtained which may compare with that of an ideally constructed pentode [5]).

Another favourable property of the powdered zirconium is its great capacity for radiating heat, which amounts to 80-90 per cent of that of a black body [6]). Because of this the specific dissipation of the anode and grids could be further considerably increased. It is clear from *fig. 8* how this permitted a reduction in dimensions compared with the earlier valves.

Thanks to the measures here described a great improvement could be obtained in the short wave properties. Most of these transmitting valves can be used on wave lengths down to 3 m without the efficiency, which amounts to 70 to 75 per cent for long waves, being too much diminished (see *fig. 9*).

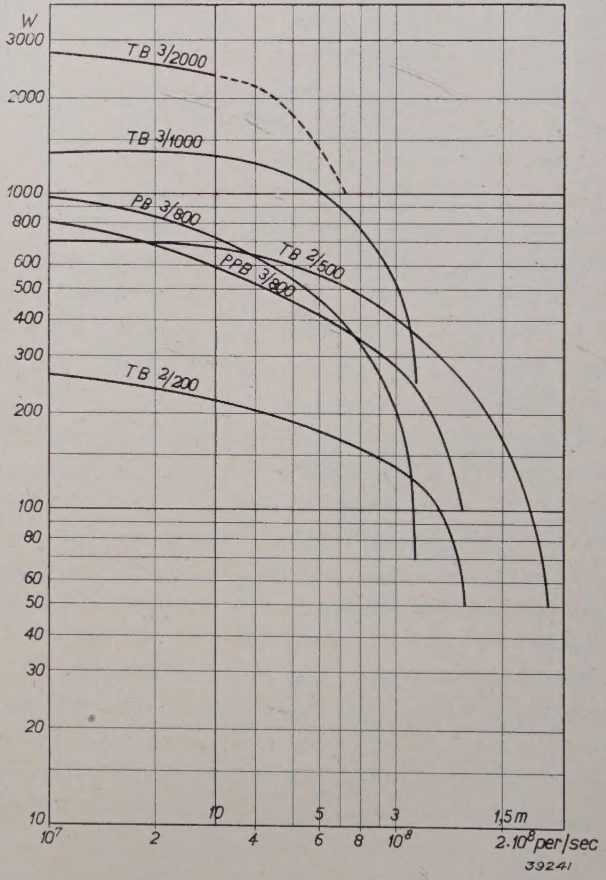


Fig. 9. Survey of the maximum obtainable outputs of the new series of transmitter valves on short waves. In different cases the triodes give a higher output than the corresponding pentodes; this takes place, however, at the expense of a much higher excitation power.

---

[5]) In judging this characteristic it must be kept in mind that different requirements are made of a transmitter valve than of a low-frequency output amplifier valve. Thus the "zirconium tetrode" when used as low-frequency amplifier will still give too much distortion, while in excitation or amplification of high-frequency energy this is of no importance.

[6]) Other finely divided substances such as tungsten also possess this property (see the article referred to in footnote [3]).

LAMPS FOR USE IN PHOTOGRAPHY

by N. A. HALBERTSMA and J. A. M. van LIEMPT.

771.448.1

In lamps for photographic purposes the temperature of the filament is much higher than in lamps for ordinary use at the expense of the life of the lamp. A much higher luminous efficiency is obtained, however, and this gain is manifested more strongly by the sensitive photographic plate than by the eye. Quantitative data are given on this subject. In conclusion the different types of electric lamps for photography by artificial light are briefly discussed.

At the present time there are many kinds of sources of light available for making photographic exposures by artificial light. Besides magnesium light, which was formerly much used for photography by artificial light, and from which the flashlight lamp was developed ¹), electric lamps are commonly used, especially in studios. They give of course less intense illumination and are also less easily moved about than the flashlight lamp, since this lamp carries its own store of energy, while the incandescent lamp must be connected to the light mains. On the other hand, however, it may be regarded as an advantage that with electric lamps the object to be photographed can be studied beforehand in the full illumination in which it is to be photographed, and that the expense of the electric lamp per exposure in less than that of a flashlight lamp. Whether the advantages of the flashlight lamp or those of the electric lamp are to predominate will depend on the purpose in view. So each of the two types of light sources has its own field of application.

In recent years electric incandescent lamps have been replaced in many fields of illumination by discharge lamps. Some of these discharge lamps, as *e.g.* the sodium lamp, exhibit unexpectedly favourable properties for photographic use ²). The incandescent lamp, however, because of the ease of use and the absence of auxiliary apparatus, will maintain its importance in this field, so that it is justifiable to manufacture lamps adapted to the special needs of photography. The following account of the development of such lamps may serve to facilitate a choice among the different types of lamp which are manufactured for photographic uses.

Actinic radiation intensity and length of life

The choice of temperature for the filament of an electric lamp represents a compromise between two requirements: that of a high luminous efficiency and that of a reasonable length of life. It is possible to make lamps of very different life, but the longer the life the lower the temperature of the filament as well as the efficiency will be. Lamps for general lighting purposes are designed for an average life of 1000 hours. The luminous efficiency then amounts to 10—16 lm/W, depending upon the size and the voltage of the lamp. The permissible load on the house fuses or group fuses does not permit generally to connect more than 2 lamps of 500 W, corresponding to a light flux of 16,000 lumens.

Although the illumination which is thus obtained does not amount to more than a few per cent of average daylight, the adapted eye will consider a room illuminated with the light flux mentioned as "bright as daylight". For photographic materials, however, where the phenomenon of adaptation is absent, the flux of light mentioned will be insufficient in many cases, especially when, for example for the sake of depth of focus, the large apertures of modern types of lenses cannot be used.

It is therefore desirable to increase for photographic purposes the efficiency of electric incandescent lamps at the expense of the length of life. Since these lamps are only used for a few minutes at a time, their life is of only secondary importance and a considerable increase of efficiency is not too dearly obtained with the reduction of life to a few hours.

The increase of luminous efficiency, when the temperature of the filament is raised, may be explained as follows: in addition to an increase of the radiation for each wave length, the spectral composition of the radiation is shifted towards shorter wave lengths (see *fig. 1a*) and since the greatest part of the emitted radiation lies in the infrared, this means that a larger part of the radiation becomes visible. Since the maximum sensitivity of photographic emulsions lies at shorter wave lengths than that of the eye, the photographic efficiency of the radiation, when the temperature is raised, will increase more rapidly than the luminous flux. This photographic action, the ac-

---

¹) For the flashlight lamp "Photoflux" see Philips techn. Rev. **1**, 289, 1936.

²) On the application of sodium lamps and other gas-discharge lamps in photography see Philips techn. Rev. **2**, 24, 1937; **3**, 91, 1938 and **4**, 27, 1939.

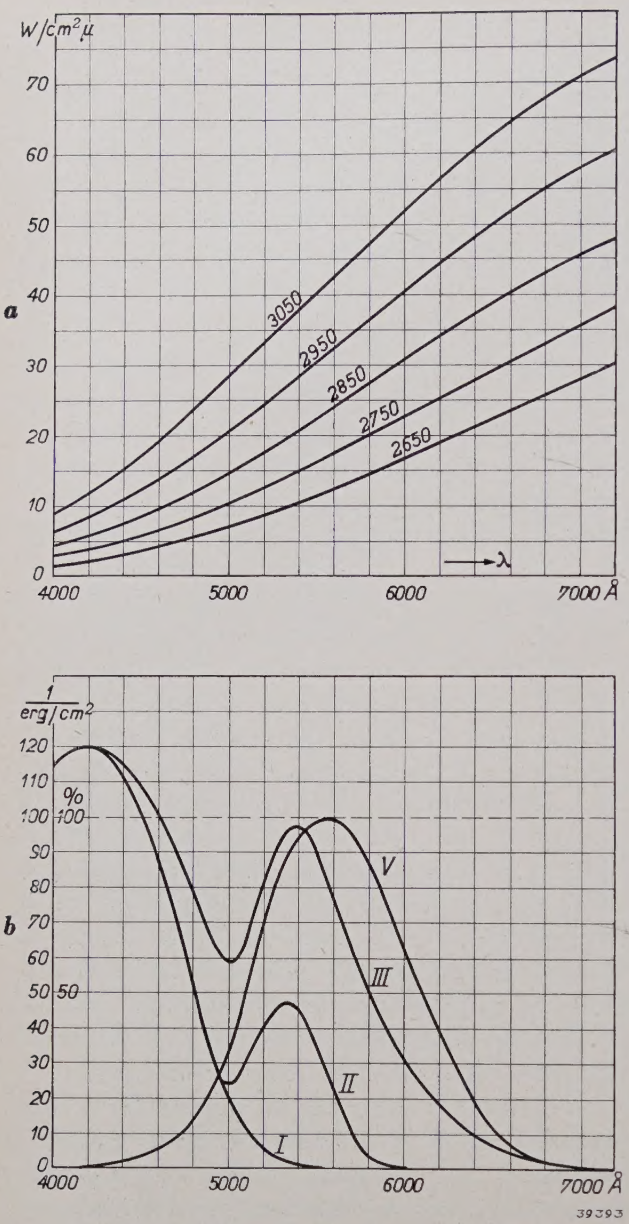


Fig. 1. *a*) Spectral intensity distribution of an incandescent tungsten surface for different temperatures which are indicated in °K.

b) Spectral sensitivity curve *V* of the eye and the sensitivity curves of different kinds of photographic emulsions (schematically): *I* non colour-sensitized, *II* orthochromatic, *III* panchromatic.

As a measure of the photographic sensitivity the reciprocal of the value of the radiation energy in erg/cm² which is necessary to obtain a density 0.1 above fog is used.

tinic value of the radiation, is of course also dependent on the nature of the light-sensitive emulsion; in the case of panchromatic emulsions the actinic value of the radiation will be much higher, and its variation with temperature will be found to correspond more closely with that of the visual efficiency than in the case of orthochromatic or ordinary, non colour-sensitized emulsions.

As an example, the average spectral sensitivity curves of a panchromatic, an orthochromatic and an ordinary, non colour-sensitized emulsion are reproduced in fig. 1*b* together with the relative eye-sensitivity curve *V*. With the help of the radiation curves of fig. 1*a* the visual and actinic efficiencies of the radiation have been calculated from these curves by a numerical integration for each temperature [3]).

The results are given in *table I*. Besides the visual intensity (light flux and three different actinic intensity values of the radiation), the table also gives the voltage, the power consumed, the life and the luminous efficiency for each temperature indicated. Since we are only interested in relative values, the values in all the columns were set equal to 100 for the normal filament temperature of 2650 °K, with the exception of the length of life, for which 1200 hours were taken [4]).

The table shows that the visual light intensity and the actinic value, like the length of life, vary very much with the temperature. If the temperature of the filament is raised by one per cent, the visual light intensity of the radiation increases by an average of 9 per cent. The actinic radiation intensities, on the other hand, rise by from 10 to 12 per cent, so that the actinic effect per unit of light flux becomes greater with increasing temperature. On the other hand, a temperature increase of one per cent causes the life to decrease by not less than 30 per cent. From this is follows that in order to achieve a reasonable improvement in actinic intensity a very large reduction of lamplife must be accepted.

The different lamps for photographic purposes

Philips lamps for photography may be divided into two classes: lamps with a life of 100 hours and lamps with a life of 2 hours. For general photographic purposes lamps with a life of 100 hours will generally be used; lamps of 2 hours life should, however, be preferred when very high intensities of illumination are required, as in making instantaneous exposures and cinematographic exposures or in working with colour film. Amateur photographers, as a rule, use these 2-hour lamps.

---

[3]) The possibility of calculating the sensitivity for heterochromatic light, starting with the sensitivity of the eye or of a photographic plate for light of different wave lengths, is based upon the addition law, see Philips techn. Rev. **6**, 1941. The fact that the addition law is really valid for photographic emulsions has been demonstrated by A. v. Kreveld, Diss. Utrecht 1933 and Physica **1**, 60, 1933.

[4]) The length of life depends not only on the filament temperature, but also upon the dimensions and the form of the incandescent filament. Therefore no generally valid conclusions can be drawn from the values in the table. The dimensions of the filament here assumed correspond approximately to those of the "Photolita" lamp, type S.

Table I

Properties of an electric lamp with tungsten filament for different temperatures *). The data for the calculation are due to C. Zwikker, Physica **5**, 252, 1925.

Temperature	Colour temperature	Voltage	Power consumed	Life	Light flux	Luminous efficiency	Actinic value of radiation (%)		
°K	°K	%	%	h	%	%	panchromatic	orthochromatic	non colour-sensitized
2 650	2 710	100	100	1 200	100	100	100	100	100
2 750	2 820	113.5	119	270	141	116	148	154	159
2 850	2 930	125.2	141	70	196	135	212	230	244
2 950	3 040	138.4	164	19	265	156	299	336	365
3 050	3 150	153	191	6	349	181	408	473	526
3 150	3 260	169	223	2	455	209	548	657	747
Average percentage increase of above quantities for 1 per cent temperature increase									
1	0.94	2.89	4.64	—31	9.0	4.3	10	11	12

*) The actinic values for the panchromatic, the orthochromatic and the non-sensitized emulsions are set equal to 100 for a temperature of 2 650 °K, and may not therefore be compared with each other. To obtain comparable figures the values for the orthochromatic and panchromatic emulsions should be multiplied by 1.57 and 3.56, respectively.

Besides the length of life of the lamp, its power consumption and light distribution are also features to be considered.

The lamps are made in sizes of 500 watts and 250 watts, for all the usual mains voltages. With a life of 100 hours lamps of 500 watts have a light flux of 11 000 lumens, while with a life of 2 hours they develop a light flux of 16 000 lumens. The latter flux is about twice as large as that which an electric lamp of normal life would radiate for the same power.

The gain actually obtained, however, is, as already stated, greater than represented by the ratio of 1 : 2, since the actinic value of the light has increased considerably. If the average of the examples given in table I (the orthochromatic type of plate) is taken as a basis for the comparison, a lamp for photographic purposes of 500 watts, 11 000 lumens has the same actinic value as a regular electric lamp of about 13 000 lumens burning at normal temperature, while the photographer's lamp of 500 W, 16 000 lumens has an actinic value equal to about 25 000 lumens of ordinary electric light. If ordinary coiled-coil lamps would be used for photography, the largest type of which has an efficiency of about 15 lumens/watt, it would be necessary to install a total of 870 watts instead of one 500 W lamp of 100 hours and of 1 670 watts instead of one 500 W lamp of 2 hours life. In the case of the lamp with the life of two hours the power consumed will therefore be reduced in the ratio 3 : 1, which means, that with a given permissible load on the mains an actinic light flux can be excited with the lamp of 2 hours which is three times as large as that obtained by coiled-coil lamps. Colours are also reproduced more faithfully by these lamps than by daylight.

The light distribution of the different types of lamps depends upon the shape of the bulb, which may be partially covered with a mirror and on the finish of the bulb (clear, frosted, opal). A certain variety of types enables the photographer to con-

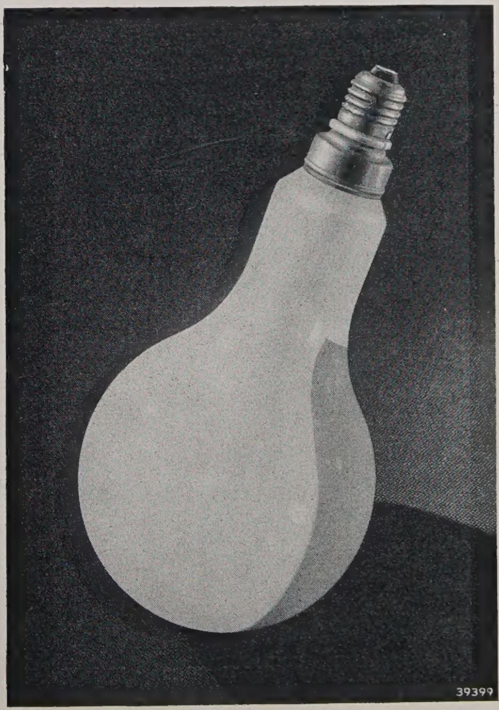


Fig. 2. The "Photomirenta" lamp with opal-glass bulb, one side of which is covered internally with a mirror.

centrate the light in various degrees depending upon the object to be photographed. By this a further appreciable gain can be achieved in the illumination of the object. We shall now briefly discuss the various types separately.

Lamps with a life of 100 working hours

The lamps with 100 working hours are manufactured in 2 types: the “Photomirenta” lamp and the “Argaphoto” lamp. Both have 500 watts, 11 000 lumens, and thus, according to the above, an average actinic intensity of 13 000 lumens. The “Photomirenta” lamp (see *fig. 2*) has a spherical opal-glass bulb 15 cm in diameter, one half of which is covered internally with a mirror. In this way a wide beam is obtained which by the diffusing effect of the opal-glass, gives no harsh shadows. In *fig. 3* the light distribution is shown. This light distribution forms a satisfactory compromise for universal application in the studio of the professional photographer.

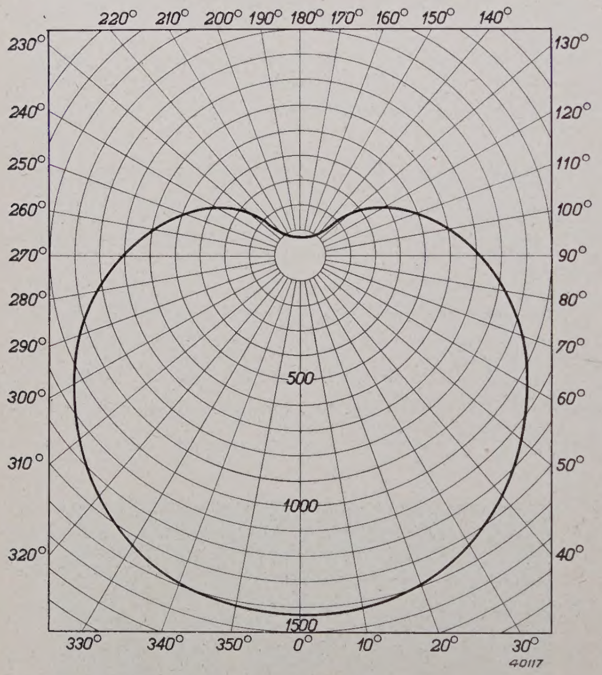


Fig. 3. Light distribution curve of the “Photomirenta” lamp (see fig. 2) in a plane perpendicular to its axis.

The “Argaphoto” lamp has a small bulb without mirror, so it should be used in combination with a reflector. In *fig. 4* the light distribution curve of a “Philiray” reflector SC 255 with “Argaphoto” lamp is shown. The maximum intensity is 20 times the mean spherical candle-power of the lamp.

The “Argaphoto” lamp with reflector is especially suitable for taking photographs with artistic effects, or for providing additional illumination to that obtained with the “Photomirenta” lamp. For infrared photography also, with or without an infrared filter, it is a very satisfactory source of light.

The lamp is furthermore important in taking photographs with less sensitive kinds of films, such as the so-called duplicate film (which can be developed directly into a positive), and the modern colour films for miniature cameras.

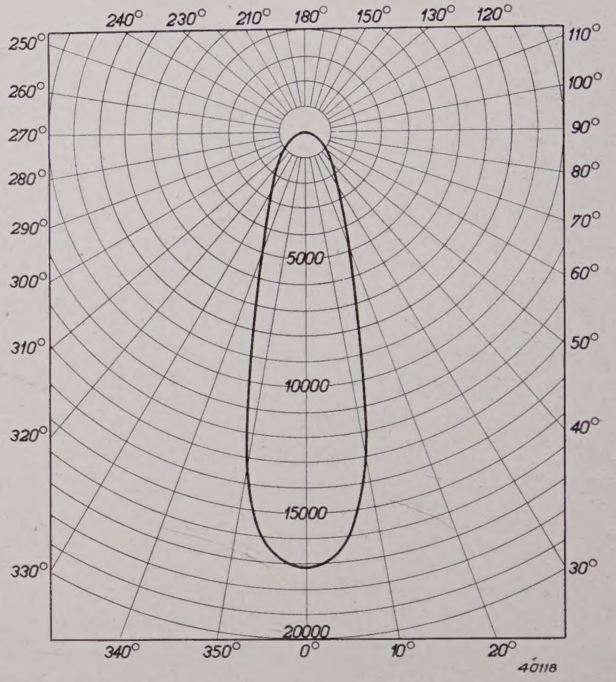


Fig. 4. Light distribution curve of the “Argaphoto” lamp with “Philiray” reflector, type SC 255.

Lamps with a life of two hours

The lamps of two hours are made in two sizes: 250 watts with a flux of 9 000 lumens, and 500 watts with a flux of 16 000 lumens.

These “Photolita” lamps, type SM and NM,

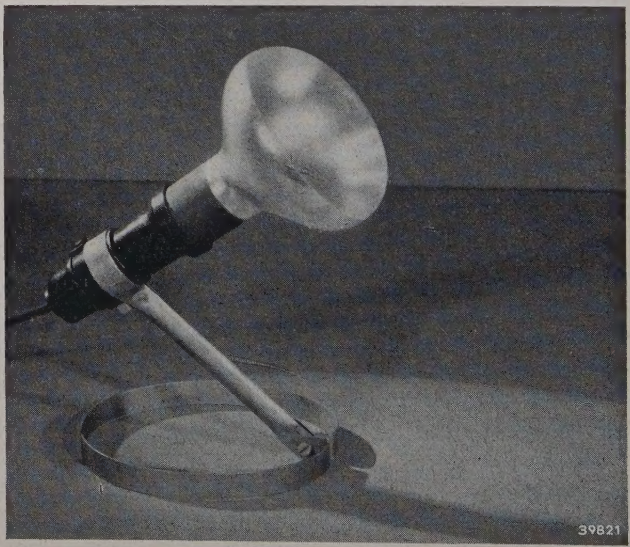


Fig. 5. “Photolita” lamp with mirrored bulb in an adjustable lampholder.

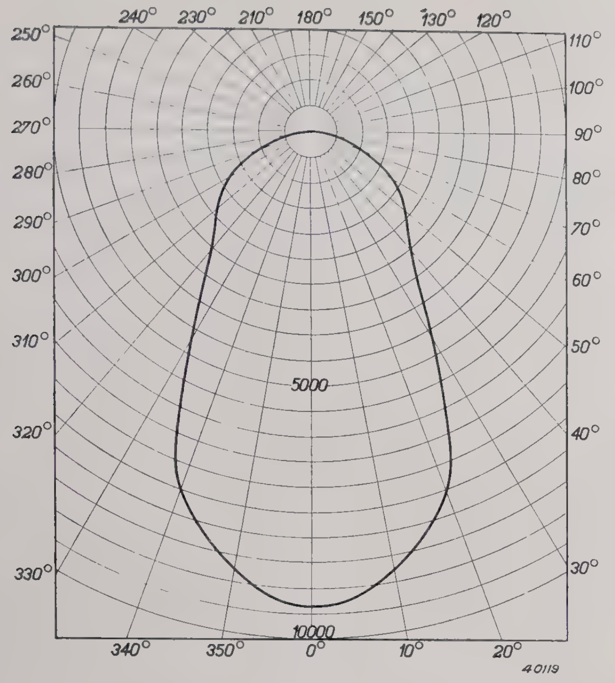


Fig. 6. Light distribution curve of the "Photolita" lamp with mirrored bulb.

have a bulb of a special shape, part of which is provided with a specular reflecting layer. The outer surface of the bulb is frosted. These lamps can therefore be used without reflector, which greatly facilitates their use by amateur photographers (see *fig. 5*). "Photolita" lamps type S (250 watts) and N (500 watts) are made without reflecting layer, the bulb being inside frosted. They should only be used with a separate reflector. *Fig. 6* presents the light distribution curve of the "Photolita" lamp type NM.

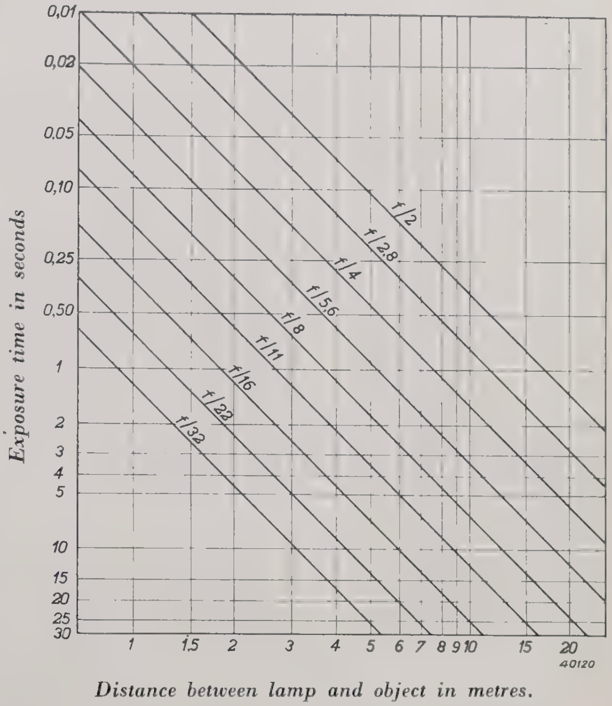


Fig. 7. Exposure diagram for a "Photolita" lamp NM or N (the latter being used in a mirrored reflector. The diagram is based on the use of orthochromatic material of 4 000 H. & D. or of panchromatic material of 2 400 H. & D.

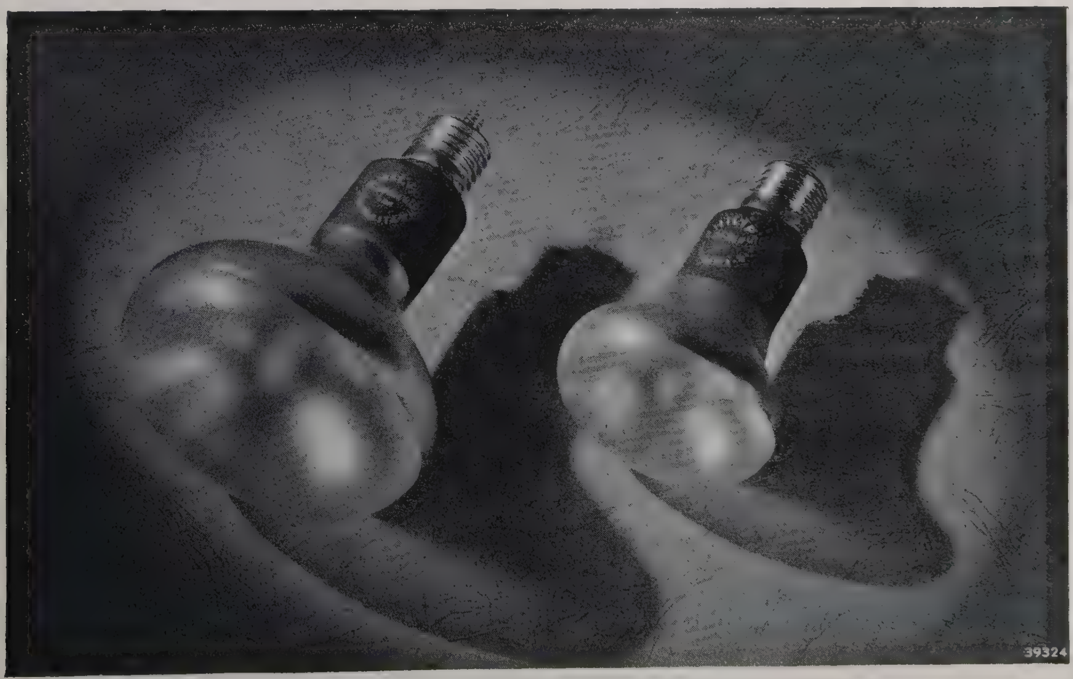


Fig. 8. The "Photolita" lamp with internal mirror. In order to prevent radiation of light toward the rear, the neck of the bulb is blackened. The base of the lamp is provided with a small spiral spring in order to make it possible to screw the lamp into the lampholder in the prescribed position.

In connection with the highly loaded filament some precautions should be taken in using the "Photolita" lamps in order not to shorten still more the already short life. Lamps with mirror must not burn in any position, but only in that indicated on the bulb of the lamp. Moreover, the mains voltage must never be higher than that marked on the lamp. Care must also be taken not to switch on the lamp for a longer time than is absolutely necessary for making the exposure. A convenient method of accomplishing this is the use of a series-parallel switch with which two lamps can be connected in series or in parallel as desired. For the arranging of the light sources, which in careful work may take considerable time, the lamps are connected in series, so that they burn at reduced voltage and have a practically unlimited life (about 1 000 hours). Shortly before the exposure the lamps are connected in parallel. The exposure diagram of *fig. 7* shows the exposure times which can be applied, when using "Photolita" lamps.

Lamps for exact colour rendering

The combination of any given light source with any kind of negative material does not in general produce a correct colour rendering, *i.e.* the blackening of the negative does not entirely correspond to the brightness values of the different colours by daylight. If an orthopanchromatic or an panchromatic emulsion is used for the negative these inaccuracies will usually not be disturbing when electric lamps are used. For very high requirements, the colour rendering can sometimes be slightly improved by the use of a blue-green or green filter.

For still more red-sensitive pan-material special lamps are made provided with an inside-frosted blue-glass bulb. There are two sizes, the photographic lamps type C of 500 watts and type E of 1 000 watts. The life of these lamps is 300 hours. These lamps can be used for extra illumination in daytime, when making colour photographs on daylight film, which does not give a correct colour rendering with the light of ordinary electric lamps without filter.

A SIMPLE SYSTEM OF BAND SPREAD IN SHORT-WAVE RECEPTION

by C. J. van LOON.

621.396.662 : 621.396.62.029.58

While in the system of band spread described previously in this periodical use was made of separate variation elements (variable condenser or coil) for the accurate tuning within each of the five or six short-wave bands, a system has now been worked out in which the ordinary rotary condenser used for normal tuning is also used for this accurate tuning. The range of variation of the circuit capacity is in this case reduced to the desired small size by the additional connection of fixed condensers. The results of this, and several particulars of the practical construction are discussed. The most important advantages of the system, in addition to a better signal-to-noise ratio and complete freedom from microphonic effect, are the simplicity of the mechanical construction and of the operation. The latter is now reduced to the same manipulation as the tuning to long-wave and intermediate-wave stations and also takes place with the same knobs.

Introduction

The resonance frequency of an oscillation circuit which is composed of a fixed self-induction coil and a rotary condenser of the usual construction generally changes between the extreme positions of the rotary condenser approximately by a factor 3.5. For the tuning of radio receiving sets, therefore, the region of the long waves (about 700-2000 m), the intermediate-wave region (60-560 m) and the short-wave region (13-50 m) can each be covered with the help of one definite coil which is connected to the oscillation circuit by means of the so-called wave-length switch.

In the region of short waves from 13 to 15 m a complete turn of the rotary condenser thus corresponds to a frequency variation of about 22 500 — 5 800 = 16 200 kc/s. If two transmitting stations in this wave-length region lie side by side with the customary interval of 10 kc/s between them, an extremely slight twist of the rotary condenser (usually 1/1 600 of the full turn) will change the tuning from the one transmitter to the other. In order to make the tuning just as easy at these short wave lengths as we are accustomed to with ordinary broadcasting waves, there would have to be an extremely fine and accurately reproducible setting of the rotary condenser as well as a very much elongated station dial.

The structural difficulties which are hereby encountered have led to a search for a solution in a different direction. This solution is based upon the fact that the broadcasting stations in the region of short waves mentioned are concentrated according to the present international agreement in only seven relatively narrow bands, see *table I.*

Table I

Band	Frequency limits in megacycles/sec	Δf in kc/s	$\Delta f/f$
13 m	21.45 - 21.75	300	1.4%
16 m	17.75 - 17.85	100	0.6%
20 m	15.10 - 15.35	250	1.7%
25 m	11.70 - 11.90	200	1.7%
30 m	9.50 - 9.70	200	2.1%
40 m	7.20 - 7.30	100	1.4%
50 m	6.00 - 6.20	200	3.3%

Each of these bands may be considered as a separate wave region analogous to the regions of long waves and intermediate waves, but with the difference that within each of the bands the frequency need not be varied by a factor 3.5 but only by a few per cent (column $\Delta f/f$ table I). Therefore one may tune to every band with the help of an absolutely constant, fixed coil and condenser, and the required slight frequency variation within the band can be obtained with an extra variable coil or condenser whose self-induction or capacity need only be a few per cent of that of the fixed elements, and therefore need not be so accurately reproducible. Each band is in this way spread over a full turn of the variation element and over a corresponding separate frequency scale with the normal density of stations. This is the origin of the common name of "band spread" for these tuning systems.

In a previous article in this periodical [1]) the principle and two different systems of band spread in practical use have already been discussed. In the case of the first system the tuning to each band was obtained by combination of certain fixed coils and condensers. The set in which this system was used was a superheterodyne receiver with three tuned circuits, namely aerial circuit, high-frequency amplifier circuit and oscillator circuit; the band spread was, however, applied only to the last circuit. In order to tune the other two circuits which work without band spread at least roughly, they first had to be set on the band with the ordinary set of rotary condensers; then by means of a switch the condenser group of the oscillator circuit was replaced by a fixed capacity with a separate small variable condenser in parallel with it. The latter had a separate drive and station dial by means of which it was now possible to tune accurately within the band.

---

[1]) Radio sets with station dials calibrated for short waves, Philips techn. Rev. 4, 284, 1939.

In the case of the second practically constructed system the same fixed coil was used for all the bands and the tuning to each band was accomplished by putting the set of variable condensers (linear action condensers) used for the ordinary tuning into a definite very accurately determined position mechanically. Thus all three circuits were already tuned to the band, and in two circuits, namely that of the oscillator and that of the high-frequency amplifier stage, band spread was now applied by tuning within each band with a small variable self-induction (coil with sliding iron core), which again had its own drive and dial.

We shall not go into the specific advantages and disadvantages of these systems; attention must only be called to the fact that the introduction of each of these band spread systems caused the appearance of two new operating knobs on the front panel of the set, namely one for the choice of the band and one for tuning within the band. Although of itself this is contrary to the universal attempt to keep the operation of receiving sets as simple as possible, it was accepted in the expectation that when new facilities are offered the user will accept a somewhat more elaborate operation into the bargain. Then after some time, when the novelty has worn off, in addition to improvements the requirement of simple operation again becomes important, while at the same time the manufacturer will also try to use the new invention in cheaper sets by simplifications in the construction, and thus make it available to a larger circle of users.

In conformance with this general line of development, a new system of band spread has been worked out by Philips, which makes the operation just as simple as it formerly was. The basic idea is that it represents a certain extravagance to use a separate variation element for the tuning within the band while there is already a variable element present in the shape of the ordinary rotary condenser. The latter has of course a range of variation of the capacity which is several hundred times as large as is required for the tuning in each band. The variation of the circuit capacity resulting from this can, however, be reduced to the desired size in a relatively simple way. The separate operation arrangements for the band spread may then disappear again: the choice of band can be made with the same wave switch and the tuning within the band with the same tuning knob which is used for the intermediate and long-wave regions [2]).

The use of the ordinary tuning condenser as variation element

How is the reduction of the variation range accomplished? More precisely stated, how can it be brought about that upon a full turn of the rotary condenser the total capacity C of the oscillator circuit varies only by a small amount ΔC, and $\Delta f/f = {}^1/_2\, \Delta C/C$ takes on the values indicated in table I (or slightly larger)?

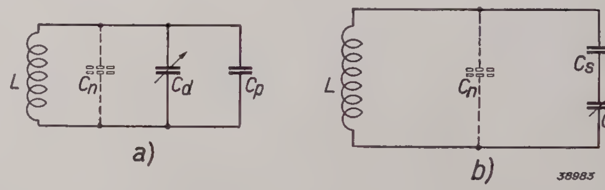


Fig. 1. In the oscillator circuit consisting of the coil L and the rotary condenser C_d a fixed condenser can be connected in parallel (C_p in *a*) or in series (C_s in *b*) with the rotary condenser. C_n represents the parasitic capacities in the circuit.

The simplest way of reducing the variation range of the capacity is to connect a fixed condenser in parallel or in series with the rotary condenser, see *fig. 1a* and *b*. Let us first consider fig. 1*a*. In this diagram C_n represents all the parasitic capacities of the circuit. Without C_p the circuit capacity would change from the small values $C_n + C_{d\,min}$ (when $C_{d\,min}$ is the zero capacity of the rotary condenser) to the value $C_n + C_{d\,max}$ upon a complete turn of the rotary condenser. In a practical case the following may be true:

$$C_n = 30\ \mu\mu\text{F},$$
$$C_{dmin} = 10\ \mu\mu\text{F},$$
$$C_{dmax} = 450\ \mu\mu\text{F}$$

so that the circuit frequency may vary by the factor $\sqrt{450/40} \approx 3.5$ already mentioned. After the connection of C_p, however, the ratio between maximum and minimum capacity becomes

$$\frac{C_p + C_n + C_{dmax}}{C_p + C_n + C_{dmin}}$$

and it is clear that when the capacity C_p in parallel is chosen large enough the frequency variation $\Delta f/f$ upon a full turn of the rotary condenser can be reduced to any desired small value. For example, in order to make $\Delta f/f = 2$ per cent in the foregoing specific example C_p would have to be 11 000 pF. If fig. 1*b* is considered in the same way it is clear that the same result can be obtained by the connection in series of a sufficiently small capacity C_s; in the example for $\Delta f/f = 2$ per cent the series capacity C_s would have to be about 4.2 pF.

---

[2]) In the first practical model of the new system, in order not to make the wave switch too elaborate or complicated, an extra switch was introduced for the choice of the band. This is, however, not essential and can be avoided by using a different switch mechanism.

In the first case therefore the circuit capacity is made very large, in the second very small. Neither of the two cases therefore is of direct practical use. The circuit capacity may not be too small, since then the parasitic capacities of coil windings and connections, which usually depend very much on the temperature and other influences, make up too large a part of the whole capacity and their variations would therefore cause the circuit capacity to vary too much. On the other hand, especially when we consider the oscillator circuit of a superheterodyne set, the circuit capacity may not be too large, since it would then become difficult to cause the circuit to oscillate. The diagram of the oscillator circuit shown in *fig. 2* illustrates this. At a certain grid A.C. voltage on the valve an anode alternating current flows. This causes an A.C. voltage on the *LC* circuit and this in turn an A.C. voltage in the back-coupling coil. This A.C. voltage must be (at least) equal to the original grid A.C. voltage in order that the oscillation shall not die out. Now this A.C. voltage will be greater the tighter the coupling of L and L_t, and the larger the impedance in parallel with the *L.C.* circuit (*i.e.* the voltage on L). This impedance is given for the resonance frequency ω by $R = Q/\omega C$, where Q is the quality factor of the coil. To make the circuit oscillate, therefore, especially on short waves, the back-coupling must be tighter the greater the capacity C. Structurally it is not so easy to realize the desired tight back-coupling, especially since a smaller coil L is again necessary for the tuning with larger C.

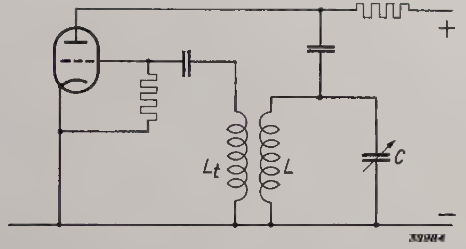


Fig. 2. Diagram showing the principle of the connections of the oscillator of a superheterodyne receiving set.

These considerations led to the requirement for the short waves here considered that the circuit capacity must not be smaller than about 150 pF or larger than about 250 pF.

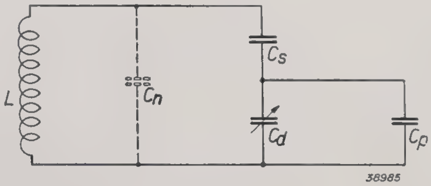


Fig. 3. Connections with which the percentage variation of the circuit capacity upon a full turn of the rotary condenser C_d can be reduced to any desired small value, while the total circuit capacity can take on a prescribed value.

This requirement, together with that of a given value of $\Delta C/C$, can be satified by connecting two fixed condensers C_p and C_s as indicated in *fig. 3* to the rotary condenser. With this complex of capacities the total circuit capacity for the minimum and maximum positions of the rotary condenser amounts to

$$C_1 = C_n + \frac{1}{\dfrac{1}{C_s} + \dfrac{1}{C_{dmin} + C_p}},$$

$$\text{and} \quad C_2 = C_n + \frac{1}{\dfrac{1}{C_s} + \dfrac{1}{C_{dmax} + C_p}}, \text{ respectively.} \qquad \ldots\ldots \quad (1)$$

We may now prescribe definite values for C_1 and C_2, *i.e.* for the circuit capacity and its variation, and we then have in (1) two equations from which the desired values of C_s and C_p can be calculated. If for example with the values of C_n, C_{dmin} and C_{dmax} we wish to obtain the values $C_1 = 162$ μμF, $C_2 = 171$ pF, we find that we must choose; $C_s = 160$ pF and $C_p = 750$ pF.

If the aim is to spread the frequency region Δf of each of the bands of table I (plus a small frequency region as reserve on either side) over the complete deviation of the rotary condenser, a different C_1 and /or C_2 must actually be prescribed for each band, since according to table I, $\Delta f/f$ which is equal to $^1/_2\ (C_1 - C_2)/C_1$ is smaller for the short waves than for the long. Now in order not to make the switch which serves for the transition from one band to the other too complicated, the same combination C_s, C_p is used for all bands, while a fairly equal spreading of the different bands is obtained by connecting some extra capacity in parallel with the whole for shorter waves (*i.e.* C_n in equation

(1) is increased). In this way the largest circuit capacity is obtained at the shortest wave length. This is also an advantage in connection with the drift of the frequency: it is exactly at the shortest waves, where the tuning is most sensitive to small capacity variations, that the parasitic capacities subject to drift have the least influence.

Even without the desirability of equal spread a small adjustment capacity (trimmer condenser) must be added for each band in parallel with C_n, in order to be able to tune the circuit accurately to the band, even with slight deviations in the capacity and self-induction values of the fixed condensers and coils used. It is of course a primary requirement that the capacity and self-induction values mentioned should drift so little when in use that after the adjustment in the factory the trimmer condenser need not be set again.

Incidentally, it may be stated that it was also desirable to leave unchanged the combination of C_s, C_p for all bands, in order to obtain the same scales for the bands for every set in series manufacture. The tuning, especially on high frequencies, is very much affected by the parasitic self-inductions of the connections between C_s, C_d and C_p, which are subject to variations. A deviation in length of 1 mm of the connecting wires (corresponding to a deviation of the self-induction of 0.001 μH) already changes the calibration of the scale for the 13 m band by 15 kc/s! Due to the fact that with fixed values of C_s and C_p the connection lines could be kept very short, it was possible to make the parasitic self-inductions sufficiently reproducible.

Other particulars of the connections

With fixed C_s, C_p the wave switch with which the bands are selected need only provide for the inclusion in the connections of the correct coil and of the supplementary capacities mentioned for each band. Since with an increasing number of positions of the switch the difficulties in its construction increase rapidly, we limited ourselves to spreading five of the bands in table I. We chose the first five since in the case of the last two it is relatively less difficult to tune in the ordinary way. Even with this limitation to five bands there occurred a difficulty, although it was not serious, in connection with finding space for the required fixed coils. The coils were mounted in standardized cans each of which can hold two coils. Each additional can means an increase of the space necessary on the chassis and in general an enlargement of the whole set. In order to keep the set as small as possible it was very desirable to use as few coils as possible. Now by choosing C_s and C_p within the available limits, so that the same coil could be used for the 30 m band as well as for the continuous tuning from 13 to 50 m, and also by using a connection in parallel of coils for the other bands, the number of coils for the whole short-wave region could be reduced to four (*i.e.* two cans).

Until now we have always been speaking of one circuit, namely the oscillator circuit, which is of primary importance for the band spread, we saw that in the system of band spread here considered the ordinary rotary condenser of the oscillator circuit is used for the tuning within the bands. Now the rotary condensers of the two other circuits (aerial circuit, high-frequency amplifier circuit) which serve for the ordinary tuning are, however, mounted on the same shaft as the rotary condenser of the oscillator circuit. Since the other circuits must also be tuned at least roughly to the bands, and since the ordinary rotary condensers can no longer be used for this, we must now provide separate tuning elements for these circuits.

This apparent complication is automatically solved when we apply band spread to the other two circuits as well. Tuning to the bands then again takes place by the switching in of fixed coils, and tuning within the band by the ordinary rotary condensers, in parallel and in series with which fixed condensers C_p and C_s, respectively, are connected in exactly the same way as in the oscillator circuit. The three switches which serve for the selection of the band in the three circuits (connection of coils, supplementary condensers, etc.), are placed on the same shaft, like the three rotary condensers. In the end, while the band spread is still better than in the earlier systems described in the article cited [1]) (it is now applied to all three circuits), the operation is nevertheless simpler, the same tuning manipulations are required as in the long-wave and intermediate-wave regions.

The application of band spread in all three circuits involves the use of a larger number of fixed coils and condensers than in the case of band spread in the oscillator circuit alone. There are, however, other important advantages besides the simplicity of operation. Without band spread, thus when the whole short-wave region must be covered with one turn of the rotary condenser, the circuit capacity must increase with the square of the wave length. The result is that, according to the above-mentioned formula $R = Q/\omega C$, there is a lower impedance in parallel with the oscillator circuit at longer waves and thus also less building up of oscillation (in the aerial circuit) or amplification (in the high-frequency amplifier stage). With band spread, however, where a different coil is used for every band, the circuit capacity can be chosen equally small in all the bands, or on longer waves (30 m) it may even be chosen smaller than in the other bands (see above), so that now a greater amplification and thus a better signal-to-noise ratio can be obtained over the whole region. This advantage is even more strongly emphasized by the fact that in the adjustment of the first two circuits such a great accurary is not required, and therefore such great care need not be taken to prevent a drift of the total circuit capacity. Because of this, with a given value of the parasitic capacities a smaller total circuit capacity is sufficient. In our case the fixed capacities C_p and C_s — in each circuit there is again only one set C_p, C_s for all five bands — have such dimensions that a total circuit capacity of about 80 pF was obtained, while without band spread the circuit capacity in the 30 m band would have had to be about 180 pF.

Summarizing it may be stated that there are numerous advantages in the system of band spread here described, although a fairly large number of fixed coils and condensers must be used. Not only is the tuning entirely insensitive to mechanical vibrations and shocks (absence of the microphonic effect) as in the case of all band spread systems with fixed condensers for the bands, but, moreover, the operation is just as simple as for the tuning on long-wave and intermediate-wave stations; furthermore no special expensive mechanical arrangements are necessary for the band spread, and the signal-to-noise ratio could be made considerably better than in the case of short-wave reception without band spread.

THE MAXIMUM ELECTRICAL FIELD STRENGTH FOR SEVERAL SIMPLE ELECTRODE CONFIGURATIONS ¹)

by A. BOUWERS and P. G. CATH.

537.212 : 621.3.027.7

The maximum voltages which may occur on parts of high-voltage apparatus determine the chance of breakdown, either toward earth or toward other points. Formulae and tables are given for different configurations which may be of service in designing high-voltage apparatus.

Introduction

Apparatus and installations have repeatedly been described in this periodical in which very high electrical voltages occur, such as X-ray tubes and generators for very high direct or surge voltages ²). In the construction of such apparatus one object is to decrease the dimensions still further while retaining the same properties. The occurrence of breakdown, which will usually be in air, when the electrical field strength at some point on the surface of one of the electrodes exceeds 30 kV/cm, determines in general how far this object may be pursued.

The smaller the dimensions of such constructions the more carefully the shape of the different components must be considered. The familiar fact that sparks jump much more easily between electrodes with small radii of curvature (points) than between electrodes with a large radius of curvature, with the plane surface as a limiting case, furnishes the simplest contribution to the knowledge which is required in all these cases.

In the articles referred to a number of methods are also discussed which may be used to avoid breakdown. For example, plane terminal electrodes are fixed to points with great curvature, and in this way the maximum field strength is decreased. For such high-voltage installations thin wires with a small radius of curvature are not used for the current supply, but thick hollow tubes which have a much lower field strength on the surface at the same potential. Components which are too sharply curved at certain points are entirely surrounded by spheres which are connected conductively to them. Since no electric field exists within the sphere, and the field strength on the outside of the enveloping sphere is lower on an average than at the sharp points of the components enveloped, breakdown takes place only at a higher potential.

In such considerations it is naturally important to be able to estimate the magnitude of the field strength in the dielectric with a chosen model. The means of doing this are in the form of calculations which are based fundamentally on Laplace's equation and on Gauss' theorem and on several simple empirical rules of calculation derived from the former. Together these allow the constructor to make a fairly sure estimate of the load on the dielectric in his design, which estimate is in this case of just as great importance as the insight into its mechanical strength. Great accuracy is here of less importance than the possibility of being able to apply the prescriptions efficiently, quickly and reliably to all the configurations which he encounters.

What electrode configurations are used in practical cases?

This question cannot be answered for all cases which occur, since each new construction may lead to different designs which have not formerly been used. If, however, we consider a fairly complicated apparatus such as the generator for 1.4 MV reproduced in *fig. 1*, which is intended for the Laboratoire de Chimie Physique in Paris, it strikes one that certain simple arrangements of spheres and cylinders are repeated over and over again. In other cases flat planes are encountered in combination with spheres and cylinders. This is not of course an arbitrarily accepted principle of construction, but the consequence of the necessity of keeping the field strength on the electrodes as small as possible with given potential differences and distances between the electrodes.

At a given distance and a given potential difference the maximum field strength on the electrodes is smallest when the field is homogeneous and the lines of force are thus parallel, as is the case between very large, charged, plane plates. Such plates, however, always have edges at which the field strength is locally much greater: it is greater the sharper the angle at which the surface passes from one direction to another. By rounding off edges, therefore, the transitions are made as gradual as possible. Curved surfaces will therefore be encountered in the construction; if they are curved in one direction cylindrical electrodes are obtained, if in two direc-

---

¹) Adapted from Chapter II of A. Bouwers, Elektrische Höchstspannungen, Springer, Berlin 1940.

²) Philips techn. Rev. **1**, 6 and 235, 1936; **2**, 161, 1937; **3**, 259, 292, 306, 331, 1938; **4**, 153, 1939; **6**, 46, 1941.

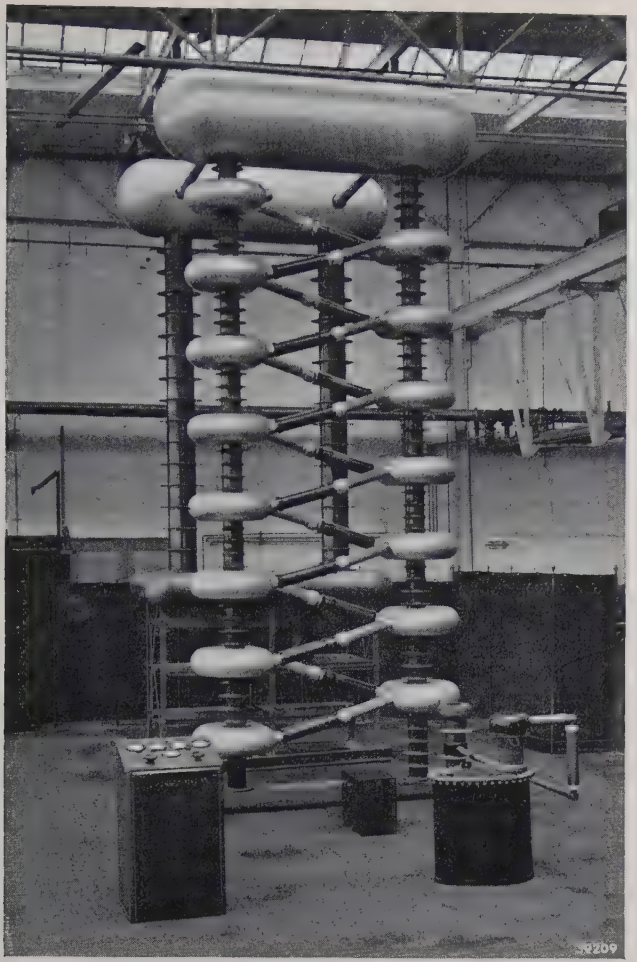


Fig. 1. Generator for 1.4 MV constructed for the Laboratoire de Chimie Physique in Paris, which exhibits the arrangement of doubly and singly curved surfaces often occurring in high-voltage constructions.

tions spherical or ellipsoidal electrodes (mushrooms). The simplest general cases with which we are concerned are therefore the following:

a) plane plates, no curvature,

b) cylindrical electrodes, curvature in one direction,

c) spherical electrodes, curvature in two directions.

Field distribution for several simple cases

In a space between charged electrodes of any given form the variation of the potential φ can in principle be calculated by solving the differential equations of the electrostatic field. This description of the electric field E (E is a vector, the component in a direction s is $E_s = -\partial\varphi/\partial s$) involves the fact that usable solutions must have a mathematical form which gives a constant value of the potential on the electrodes. For electrodes of any given shape it is difficult to satisfy this condition mathematically; in general it is easier to find a solution of a differential equation than to satisfy fairly complicated boundary conditions.

Even for very simple surfaces: plane plate, sphere or cylinder, elementary considerations are only possible when the second electrode is also a plane plate parallel to the first, a concentric sphere or a cylinder with the same axis, respectively. It is already difficult to calculate the field distribution between two charged plane plates which are not parallel. For two charged spheres which are not concentric and one of which surrounds the other the calculation of the field distribution is also not very simple. Fortunately the cases: parallel plane plates, concentric spheres and coaxial cylinders, are so important practically that we can carry out the main part of our task. We shall begin by giving briefly the results confining ourselves to the field in air (see later for other dielectrics).

1) With two parallel plane plates at a distance a from each other the potential φ which satisfies Laplace's equation

$$\frac{\partial^2\varphi}{\partial x^2} + \frac{\partial^2\varphi}{\partial y^2} + \frac{\partial^2\varphi}{\partial z^2} = 0 \quad . \; . \; . \; . \quad \text{(L)}$$

and the boundary conditions is:

$$\varphi = Ax + B,$$

where x is measured according to the normal and A and B are constants.

If $\varphi = \varphi_1$ for $x = 0$ and $\varphi = \varphi_2$ for $x = a$ one finds that

$$A = \frac{\varphi_2 - \varphi_1}{a} = -\frac{U}{a},$$

where $U = \varphi_1 - \varphi_2$. The field strength is

$$E_x = -\frac{\partial\varphi}{\partial x} = -A = \frac{U}{a}.$$

It is constant and the field is thus homogeneous.

2) The field of a charged sphere (charge q) is easily calculated according to Gauss' theorem, which relates to a closed surface s which encloses a charge q, and is as follows:

$$\int E_n \, ds = \int \frac{\partial\varphi}{\partial n} \, ds = 4\pi q, \quad . \; . \; . \quad \text{(G)}$$

where n is the direction of the normal to the surface. With this one finds for the radial field

$$4\pi r^2 E_r = 4\pi q, \; E_r = \frac{q}{r^2}.$$

3) In the case of two concentric spheres one finds from the foregoing that $\varphi = A/r + B$, where A and B are constants and r is measured from the common centre. If the radii of the two spheres

are r_1 and r_2 ($r_2 > r_1$) and φ_1 and φ_2 are their potentials, then

$$\varphi_1 - \varphi_2 = A\left(\frac{1}{r_1} - \frac{1}{r_2}\right) = U,$$

so that

$$E_r = -\frac{\partial \varphi}{\partial r} = \frac{U}{r^2}\,\frac{r_1 r_2}{r_2 - r_1}\,.$$

The greatest value of E_r is found at the surface of the inner sphere ($r = r_1$). It amounts to:

$$E_1 = \frac{U}{r_1}\,\frac{r_2}{r_2 - r_1} = \frac{U}{a}\,\frac{r_2}{r_1}, \quad . \; . \; . \quad (1)$$

where $a = r_2 - r_1$ represents the distance between the surfaces of the two spheres. The maximum field strength is thus r_2/r_1 times as large as in the corresponding case of two parallel plates at a distance a apart. This result can also be represented by means of an efficiency factor η. This quantity is defined as the quotient of the electrical field strength E between two parallel plates at a distance a apart and the maximum electrical fieldstrength E_m in the case of non-planar conductors with a least distance a apart at the same potential difference U, so that

$$\eta = \frac{E \text{ (// plates; distance } a)}{E_m \text{ (conductors with least distance apart } a)}$$

In our case therefore

$$\eta = \frac{r_1}{r_2} = \frac{r_1}{r_1 + a} < 1.$$

The efficiency factor approaches unity when r_1 becomes infinite, a being kept constant, and it tends to zero as r_1 approaches zero. This value of η will be found in column (a) of *table I*. Its reciprocal $1/\eta$ (in our case r_2/r_1) indicates how many times greater the maximum field strength is than in the corresponding case of parallel plates at the same distance from each other.

The question may also be put as to how E_1 varies when r_2 and U are kept constant and r_1 is considered variable (a is of course not constant here). E_1 then has a minimum for $r_1 = {}^1/_2\, r_2$. The value of E_1 in that case is

$$\frac{2U}{r_1} = \frac{2U}{a},$$

while $\eta = 0.5$.

4) In the same way as in case 2) one finds for the field around a charged cylinder

$$2\pi r E_r = 4\pi q, \quad E_r = \frac{2q}{r},$$

where q represents the charge per cm length.

Table I

Efficiency factor $\eta = E/E_m$ for fields on a sphere surrounded by a concentric sphere, compared with the same factor for a sphere and a plane plate and for two similar spheres at a distance from each other.

$\frac{r+a}{r}$	concentric spheres (a)	sphere and plane plate			two similar spheres	
		rigorous (b)	$k = \frac{(a)}{(b)}$	$k = 0.9$ (c)	rigorous (d)	$k = 0.9$ (e)
1.5	0.667	0.732	0.91	0.74	0.850	0.88
2	0.500	0.563	0.89	0.555	0.732	0.74
3	0.333	0.372	0.90	0.37	0.563	0.575
4	0.250	0.276	0.91	0.275	0.450	0.45
5	0.200	0.218	0.92	0.22	0.372	0.37
6	0.167	0.178	0.93	0.185	0.318	0.31
7	0.143	0.152	0.94	0.16	0.278	0.275
8	0.125	0.133	0.94	0.14	0.244	0.245
9	0.111	0.117	0.95	0.12	0.218	0.22
10	0.100	0.105	0.96	0.11	0.197	0.20
15	0.066	0.068	0.97	0.075	0.133	0.14

(a) is calculated according to formula (1), (b) and (d) according to Maxwell, (c) according to formula (6) and (e) according to formula (8).

5) As in case 3) it may be derived from this for two coaxial cylinders ($r_2 > r_1$) that

$$\varphi = A \ln r + B,$$

so that $\varphi_1 - \varphi_2 = A \ln r_1/r_2 = U$

and $E_r = -\dfrac{\partial \varphi}{\partial r} = \dfrac{U}{r}\,\dfrac{1}{\ln (r_2/r_1)}\,.$

The largest value of E_r again occurs at the surface of the inner cylinder. One finds there

$$E_1 = \frac{U}{r_1}\,\frac{1}{\ln (r_2/r_1)} \quad . \; . \; . \; . \; . \quad (2)$$

For the efficiency factor one finds

$$\eta = \frac{r_1}{a} \ln \frac{r_1 + a}{r_1}.$$

Here also η approaches zero when r_1 approaches zero, a being constant, and η tends to unity as r_1 approaches infinity.

Furthermore one can again ascertain the value of r_1 for which the maximum field strength $E_m = E_1$ has its smallest value, with r_2 and U being kept constant. This is now the case for $r_2/r_1 = e = 2.718$. E_1 then has the value

$$\frac{U}{r_1} = 1.718\,\frac{U}{a},$$

while $\eta = \dfrac{1}{1.718} = 0.58.$

Approximation for other electrode configurations

If, continuing in this way, an attempt is made to calculate the fields for other configurations of charged electrodes, in the case of rigorous solutions the difficulties are encountered which were discussed at the beginning of the foregoing section. For example in the case of a sphere or a cylinder with a plane plate, the potential on the sphere and on the plate has a given value, and it is clear that the solutions found: $\varphi = A/r + B$ and $\varphi = A \ln r + B$ on the plate, for which r changes from point to point can never be satisfactory. By means of certain devices an adaptation of the solution to the boundary conditions is sometimes successful, and very much work has been done in this direction, although the mathematical devices rapidly become more elaborate than is desirable for practical work. For this reason the practical constructor makes little use of these results as a rule.

It is, however, possible to discuss in a simple way several important cases which give usable results on the basis of those already obtained, by means of suitable appoximations. *Such approximations are possible because in general with a given potential difference between two or more electrodes the maximum field strength occurring at the surface of one of the electrodes is not determined, or if so to only a reasonably small degree, by the shape of the other electrodes.* Because of this, with a given potential difference the maximum field strength on a cylinder coaxial with another cylinder does not differ too much for example from the field strength of the cylinder with a plane plate. This is shown in *table II* where E again represents the field strength in the case of plane plates a distance a apart and E_m the maximum field strength in the cases being compared with it.

Table II

Efficiency factor $\eta = E/E_m$ for fields on a cylinder surrounded coaxially by a second cylinder, compared with a cylinder with a plane plate and with parallel cylinders at a distance from each other.

$\frac{r+a}{r}$	coaxial cylinders (a)	cylinder and plane plate			parallel cylinders	
		rigorous (b)	$k = \frac{(a)}{(b)}$	$k = 0.9$ (c)	rigorous (d)	$k = 0.9$ (e)
1.5	0.811	0.861	0.94	0.90	0.924	0.96
2	0.693	0.760	0.92	0.77	0.861	0.90
3	0.549	0.623	0.88	0.61	0.760	0.77
4	0.462	0.533	0.86	0.52	0.682	0.68
5	0.402	0.468	0.86	0.45	0.623	0.61
6	0.358	0.419	0.86	0.40	0.574	0.56
7	0.324	0.380	0.85	0.36	0.533	0.52
8	0.297	0.349	0.85	0.33	0.497	0.485
9	0.275	0.323	0.85	0.31	0.468	0.45
10	0.256	0.301	0.85	0.285	0.442	0.43
15	0.193	0.228	0.85	0.215	0.349	0.33

(*a*) is calculated according to formula (2), (*b*) and (*d*) according to a rigorous method, (*c*) according to formula (3) and (*e*) according to formula (4).

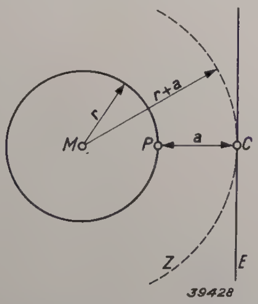


Fig. 2. The field strength on a sphere and a cylinder opposite a plane plate E has a maximum value at the point P.

In *fig. 2* this coaxial cylinder is drawn as a dotted line through point C of the plane plate. The plane plate was an equipotential plane, and it is therefore clear that if it is replaced by the coaxial cylinder a higher value will be found for the field strength on the inner cylinder. The surface elements of the outer cylinder lie closer to the inner cylinder than do those of the plane plate. Upon a comparison with the results of the rigorous calculation, however, it is found that the differences in the field strengths between the two coaxial cylinders and between a cylinder and the plate which is tangent to Z at C are relatively small. *Table II* gives the results. As was to be expected, in the case of plane plate and cylinder the field strengths are somewhat lower than for the substitution arrangement of two coaxial cylinders. The difference is, however, slight. The agreement becomes still better when a correction factor $k < 1$ is introduced into the formula. This is also shown in table II. This factor should increase slowly with increasing value of r with a constant, and for $r \gg a$ it should approach unity, since the field is then practically homogeneous. For practical purposes we choose the value 0.9. In this way one finds for the maximum field strength on the cylinder with radius r at a distance a from a flat plane

$$E_m = \frac{9}{10} \cdot \frac{U}{r \ln \frac{r+a}{r}} \quad \cdot \cdot \cdot \cdot \quad (3)$$

For two parallel cylinders both with radius r at a distance a from each other, one of which is at the potential φ_1 and the other at φ_2 the maximum field strength on one of the two cylinders can be approximated by a similar line of reasoning. The plane which bisects perpendicularly the shortest distance between them is an equipotential plane with the potential $(\varphi_1 + \varphi_2)/2$. This plane lies at a distance $a/2$ from one cylinder. If one imagines this plane again replaced by a coaxial cylinder at the potential of the equipotential plane in question, one finds for the maximum field strength on the cylinder:

$$E_m = \frac{9}{10} \frac{\varphi_1 - \varphi_2}{2} \cdot \frac{1}{r \ln \frac{r + a/2}{r}} = \frac{9}{20} \frac{U}{r \ln \frac{r + a/2}{r}} \cdot \quad (4)$$

Let us consider the case of two cylinders which are not parallel but perpendicular to each other. Apparently the maximum field strength, which produces breakdown, occurs at the surface of the smaller cylinder (radius r), at the point where the distance between the cylinders is shortest (P_1). It has been found experimentally that the potential difference at which breakdown occurs in the case of unlike mutually perpendicular cylinders at a distance a from each other (*fig. 3*) does not differ very much from the case of parallel cylinders at a distance a from each other. Thus for crossed cylinders at a distance $P_1P_2 = a$ it is possible to write for the greatest field strength that occurs on the cylinders, as an approximation:

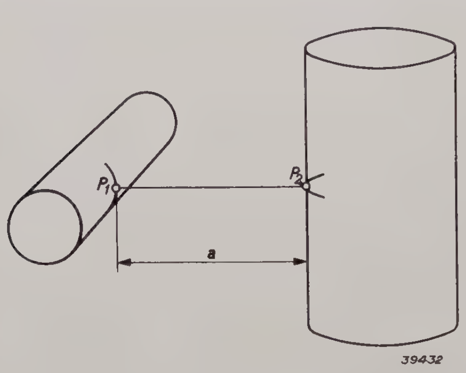


Fig. 3. Two mutually perpendicular cylinders have the greatest field strength on the cylinder with the smaller radius at point P_1 which lies closest to the second cylinder. This field strength is of approximately the same value as that for two parallel cylinders at the point of intersection of a line perpendicular to the axes with the cylinder of smaller radius.

$$E_m = \frac{9}{20} \cdot \frac{U}{r \ln \frac{r + a/2}{r}} \quad . \; . \; . \; . \quad (5)$$

It is obvious from the foregoing that in the case of a sphere (radius r) at a distance a from a plane plate the following may be written for the maximum field strength on the sphere:

$$E_m = \frac{U}{a} \frac{r + a}{r}.$$

The accuracy of this approximation can be verified in table I, columns (a) and (b). As shown in column (e) the agreement can here also be made better by introducing a constant correction factor k, and in this case also 0.9 is seen to be good average value for the constant, so that for a sphere with a plane we may write

$$E_m = \frac{9}{10} \frac{U}{a} \frac{r + a}{r} \quad . \; . \; . \; . \; . \; . \; . \; . \quad (6)$$

Two spheres both of radius r at a distance a from each other may again in a similar way be derived from the case of a sphere and a plane at a distance of $a/2$. Corresponding to (6) one then immediately obtains

$$E_m = \frac{9}{10} \frac{U}{a} \frac{r + a/2}{r} \quad . \; . \; . \; . \; . \quad (7)$$

For the usefulness of this approximation see table I columns (d) and (e).

The field distribution between two rings (tori) lying in the same plane may be derived from that between two parallel cylinders, like that of a ring and a cylinder whose axes are parallel.

The maximum field strength in the second case, when the cylinder and ring have the same radius, is on the ring, since the ring exhibits curvature in two directions at the point where the distance to the cylinder is smallest.

If the distance from ring to cylinder or from ring to ring is a, while the radius of both is r, we find:

$$E_m = \frac{U}{2r \ln \frac{r + a/2}{r}} \quad . \; . \; . \; . \quad (8)$$

No correction need be introduced here since due to the double curvature of the ring the field strength on it is greater than on the cylinder.

The case of a cylinder, for instance a wire which passes through a circular opening, can be derived from that of a ring through which a coaxial cylinder passes. It is always possible and also desirable to round off the edge of the opening so much that the maximum field strength occurs at the cylinder which thus has the smaller radius and the greater curvature. If one imagines the ring which encloses the cylinder to be replaced by a coaxial cylinder with the radius of the inside of the ring, one again obtains

$$E_m = \frac{U}{2r \ln \frac{r + a/2}{r}} \quad . \; . \; . \; . \quad (9)$$

Another method of approximation leads to the formula

$$E_m = \frac{U}{r \ln \dfrac{r+a}{r}}, \quad . \; . \; . \; . \quad (10)$$

which corresponds to (9) for $a \ll r$.

Influence of irregularities on the surface of the electrodes

In all the cases discussed until now the surfaces of the electrodes must be absolutely smooth. A local depression or projection leads to changes in the field strength. In a first approximation the effects of such irregularities can be seen by calculating the field strength in the neighbourhood of a hemispherical projection on a flat plane at a great distance from which there is a parallel plane. To do this we first calculate the disturbance caused by a metal sphere placed in a homogeneous field.

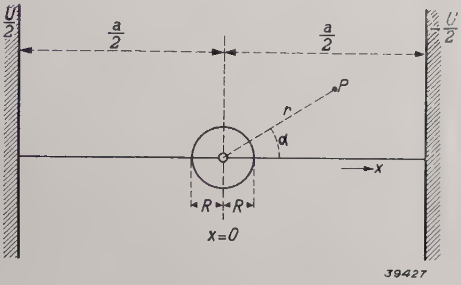


Fig. 4. A conducting sphere in a homogeneous field with the field strength E between two plane plates at potentials $-U/2$ and $+U/2$ has a maximum field strength of $3E$ on its surface.

It can be shown that upon placing a conducting sphere of radius R in a homogeneous electric field (*fig. 4*) the potential is given by

$$\varphi = -Ex + E \frac{R^3}{r^3} x. \quad . \; . \; . \; . \quad (11)$$

E is here the field strength, $-Ex$ the potential of the homogeneous field between two plates at distances $+a/2$ from the centre of the sphere. In this a is considered to be large compared with the radius R of the sphere; $\varphi = +U/2$ for $x = -a/2$, and $\varphi = -U/2$ for $x = +a/2$. The second term in formula (11) is equal to the potential of a dipole imagined to be situated at the centre [3]) which has its positive pole toward the right along the x-axis and which has a moment of R^3E. It is easily seen that for $r = R$ the potential $\varphi = 0$, *i.e.* constant, while for $r \gg R$ only the homogeneous field remains. The solution thus satisfies the boundary conditions and of course Laplace's equation.

The lines of force strike the metal surface perpendicularly. It is therefore sufficient to calculate the radial force E_r at the point $r = R$. If we set $x = r \cos a$, then

$$E_r = -\frac{\partial \varphi}{\partial r} = E \cos a + \frac{2\,ER^3}{r^3} \cos a \quad . \quad (12)$$

or, for $r = R$:

$$E_r = 3E \cos a.$$

The greatest field strength thus occurs where $a = 0$ or 180°. It amounts to

$$E_m = 3E.$$

It is thus clear that due to the presence of the sphere in the field the maximum field strength is increased by a factor 3. The calculation also holds when only the right-hand half of fig. 4 is considered and the plane $x = 0$ is considered to be conducting. For a spherical irregularity on a charged plane plate from which lines of force emerge, the maximum field strength will therefore also become greater by a factor 3. Every irregularity, even though it is not truly spherical, gives a similar increase. Thus for a semicylinder on a plane an increase of the maximum field strength by a factor 2 has been calculated. Such an increase may be expected in the case of an elongated irregularity (scratches). We give only a diagram (*fig. 5*) of the field distribution for a spherical irregularity; that for a cylindrical irregularity shows many similarities.

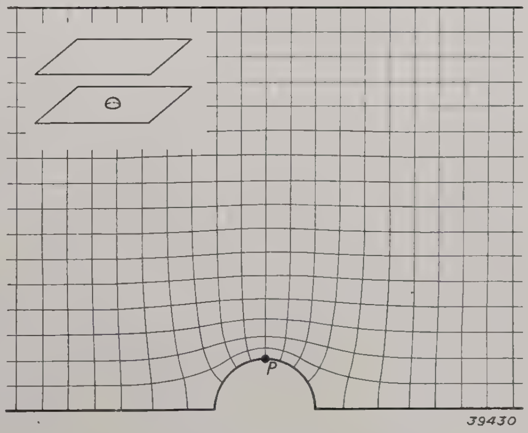


Fig. 5. Curvature of equipotential lines and lines of force in the field between two parallel plates when there is a projection in the shape of a hemisphere on one plate. At point P there is a field strength three times as large as in the homogeneous field. In the case of a half cylinder on a plate the maximum field strength at P is twice as great as in the homogeneous field.

---

[3]) By this is meant a charge q at the point $x = \delta$, combined with a charge $-q$ at $x = -\delta$, where $\delta \ll R$. The moment of the dipole is $2q\delta$.

The role of the dielectric

Until now we have not spoken of the rôle of the dielectric, since our calculations were always carried out with a dielectric constant 1. If in the problems discussed all of the space were filled with a substance with the dielectric constant ε, the field strength would not be changed for the same potential distribution. The dielectric displacement $D = \varepsilon E$ is, however, in this case everywhere increased by a factor ε. Corresponding to this increase is a change in capacity by a factor ε.

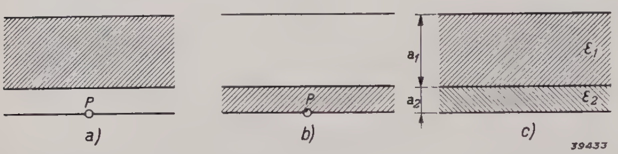


Fig. 6. Different cases of media in which the dielectric constant is not everywhere equal to unity.

If, however, space is only partly filled with a substance with a dielectric constant differing from unity, the field strength changes. Different cases hereby occur. We shall here confine ourselves to those in which the boundary surfaces are parallel plane plates (*fig. 6*) or concentric cylinders (*fig. 7*).

a) In the neighbourhood of a point P (see fig. 6) let $\varepsilon = 1$. If $\varepsilon > 1$ everywhere else, then for a dielectric bounded by plane plates the field strength at P is ε times as great, because the dielectric displacement along the lines of force is constant. A related case occurs when there are air bubbles in an insulator, for instance in the oil in a transformer. Due to the spherical form, however, the factor by which the field strength is multiplied is only $3\varepsilon/(1 + 2\varepsilon)$. Due to the greater field strength breakdown may occur in the air bubbles, accompanied by ionization, and thus depreciation of the oil.

b) In the converse case where $\varepsilon > 1$ in the neighbourhood of P and $\varepsilon = 1$ in by far the largest part of space, the dielectric displacement D changes only slightly and E_1 in the insulator becomes a factor ε smaller. The field strength E_2 in the rest of space changes only slightly.

c) In the general case of two insulators with $\varepsilon = \varepsilon_1$ and $\varepsilon = \varepsilon_2$ the capacity per cm² is determined by the two capacities C_1 and C_2 according to the formula

$$1/C = 1/C_1 + 1/C_2,$$

where $C_1 = \varepsilon_1/4\,\pi a_1$ and $C_2 = \varepsilon_2/4\,\pi a_2$.

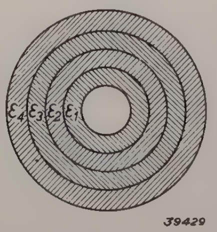


Fig. 7. Different dielectrics in coaxial layers.

The voltages on the layers are distributed inversely proportional to the capacities, thus

$$U_1 : U_2 = C_2 : C_1 = \varepsilon_2 a_1 : \varepsilon_1 a_2.$$

If we assume that $U = U_1 + U_2$, then

$$U_1 = U \frac{a_1\,\varepsilon_2}{a_1\,\varepsilon_2 + a_2\,\varepsilon_1} \quad \dots \quad (13)$$

and

$$U_2 = U \frac{a_2\,\varepsilon_1}{a_1\,\varepsilon_2 + a_2\,\varepsilon_1} \quad \dots \quad (14)$$

The field strengths in the layers are ($U/a = E$)

$$E_1 = \frac{U\varepsilon_2}{a_1\,\varepsilon_2 + a_2\,\varepsilon_1} \text{ and } E_2 = \frac{U\varepsilon_1}{a_1\,\varepsilon_2 + a_2\,\varepsilon_1} \quad \dots (15)$$

If $\varepsilon_2 = 1$, the field strength in the two layers becomes

$$E_1 = \frac{U}{a_1 + a_2\,\varepsilon_1} \text{ and } E_2 = \frac{U\varepsilon_1}{a_1 + a_2\,\varepsilon_1} \quad \dots (16)$$

The greatest field strength is E_2. For $a_1 \gg a_2$, E_2 becomes equal to $U\varepsilon_1/a_1$, for $a_1 \gg a_2$, $E_2 = U/a_2$. These are the cases discussed under *a*) and *b*).

In more complicated cases of stratified insulators between parallel plane conductors the value of E may be found by remembering that everywhere $D = \varepsilon E$ and that the voltages are distributed over the different layers according to the capacities. The determination of the field strength then always takes place in the way indicated for case *c*).

Several interesting configurations deserve closer consideration. In fig. 7 the case of concentric cylinders is represented where the dielectric is divided into layers. We have seen that the field strength increases as $1/r$ from the outside inwards. If the values $\varepsilon_1, \varepsilon_2, \varepsilon_3 \dots \varepsilon_n$ are chosen so that $\varepsilon_1 r_1 = \varepsilon_2 r_2 = \dots \varepsilon_n r_n$, the field strength in the layers is about equal. The condensers formed by the different layers then have the same capacities per unit of surface:

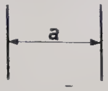
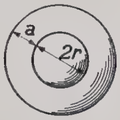
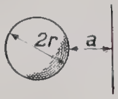
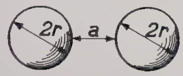
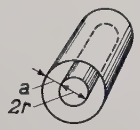
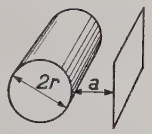
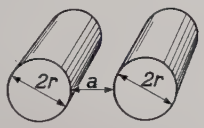
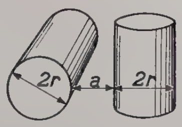
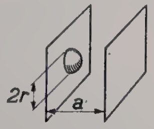
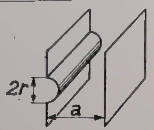
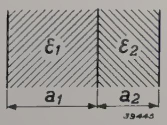
$$\frac{2\pi r\varepsilon}{4\pi d} = \frac{r\varepsilon}{2d},$$

where d is the thickness of layer. The same voltages thus act. Such an arrangement of dielectrics is often proposed for cables.

Summarizing it may be stated that by the introduction of insulators into an electric field the maxi-

Table III

Maximum field strength E with a potential difference U between the electrodes, for different electrode configurations.

Configuration		Formula for E	Example
Two parallel plane plates		$\frac{U}{a}$	$U = 100$ kV, $a = 2$ cm, $E = 50$ kV/cm.
Two concentric spheres		$\frac{U}{a}\cdot\frac{r+a}{r}$	$U = 150$ kV, $r = 3$ cm, $a = 2$ cm, $E = 125$ kV/cm.
Sphere and plane plate		$0.9\,\frac{u}{a}\cdot\frac{r+a}{r}$	$U = 200$ kV, $r = 5$ cm, $a = 8$ cm, $E = 58.5$ kV/cm.
Two spheres at a distance a from each other		$0.9\,\frac{U}{a}\cdot\frac{r+a/2}{r}$	$U = 200$ kV, $r = 5$ cm, $a = 12$ cm, $E = 33$ kV/cm.
Two coaxial cylinders		$\frac{U}{2.3\;r\;\lg\frac{r+a}{r}}$	$U = 100$ kV, $r = 5$ cm, $a = 7$ cm, $E = 22.9$ kV/cm.
Cylinder parallel to plane plate		$0.9\,\frac{U}{2.3\cdot r\;\lg\frac{r+a}{r}}$	$U = 200$ kV, $r = 5$ cm, $a = 10$ cm, $E = 32.8$ kV/cm.
Two parallel cylinders		$0.9\,\frac{U/2}{2.3\;r\;\lg\frac{r+a/2}{r}}$	$U = 150$ kV, $r = 6$ cm, $a = 20$ cm, $E = 11.5$ kV/cm.
Two perpendicular cylinders		$0.9\,\frac{U/2}{2.3\;r\;\lg\frac{r+a/2}{r}}$	$U = 200$ kV, $r = 10$ cm, $a = 10$ cm, $E = 22.2$ kV/cm.
Hemisphere on one of two parallel plane plates		$\frac{3U}{a};\ (a \gg r)$	$U = 100$ kV, $a = 10$ cm, $E = 30$ kV/cm.
Semicylinder on one of two parallel plane plates		$\frac{2U}{a};\ (a \gg r)$	$U = 200$ kV, $a = 12$ cm, $E = 33.3$ kV/cm.
Two dielectrics between plane plates ($\varepsilon_1 > \varepsilon_2$)	 ε_1 ε_2 a_1 a_2 39445	$\frac{U\,\varepsilon_1}{a_1\,\varepsilon_2 + a_2\,\varepsilon_1}$	U=200 kV, ε_1=2, ε_2=4, a_1=6 cm, a_2=5 cm, $E = 11.8$ kV/cm.

mum field strength which existed in air is considerably increased, and in extreme cases (thin layer of air) proportionally to ε, but in general less.

Examples

In *table III* a number of cases are collected of configurations of electrodes which occur commonly in the construction of apparatus for high voltages.

In addition to the formula valid for the configuration an example is given for all cases. All the formulae are derived in Briggs logarithms (to the base ten) since many persons are more accustomed to their use than to the use of tables of Napierian logarithms. We had this chart in mind when we spoke of the prescriptions which may be used to guide the constructor. Given a definite structural design, it is possible with the help of this table to calculate for every point the maximum electrical field strengths occurring from the known dimensions and potentials of the structural components. A comparison of this field strength with the breakdown voltages of the surrounding dielectrics shows the constructor what are the dangerous spots in his design. Devices for the improvement of such spots are, in addition to changes in shape (consisting for example in the increase of the radii of curvature of the components), a possible replacement of the dielectric by a different one according to the rules given above. Sometimes, as already mentioned, the electrode can be surrounded by one with a larger radius of curvature.

Although the chart given furnishes an idea of the maximum field strength occurring for a large number of shapes of electrodes, it is by no means complete. Even the case of two parallel plane plates, where $E = U/a$ can, moreover, only be correct when the plates extend to infinity. If the edges are not well rounded the first breakdown occurs at that spot. This can be avoided by choosing curved instead of plane surfaces, so that their distance apart becomes greater toward the edges [4].

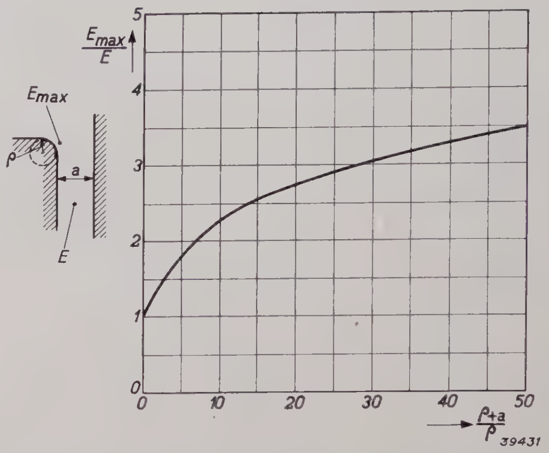


Fig. 8. Plane plate with bent edge opposite an infinite plane plate. Influence of the curvature of the bent edge on the maximum field strength.

A problem which will consequently sometimes occur in practical cases is the field distribution of a round corner opposite a plane plate. The solution of this is also by no means simple [5]. Because of the importance of this case we give the solution of this problem in a graph (*fig. 8*) which indicates values of $E_{\max}/E$ for different values of $(\varrho+a)/\varrho$ (a = distance between the plates, ϱ = radius of curvature of the rounded edge). $E = U/a$ is here the field strength between the plates, and it may be seen from the graph that for small values of ϱ, $E_{\max}$ may be more than 3.5 times as large as E.

---

[4] W. Rogowski and H. Rengier, Arch. Elektrotechn. **16**, 73, 1926.

[5] A. Dreyfus, Arch. Elektrotechn. **13**, 123, 1924, see fig. 10.

EXPERIMENTS ON THE AUSTEMPERING OF STEEL

by J. G. C. STEGWEE.

621.785.6

In the process of hardening steel the cooling of the heated object must take place so rapidly that the austenite (unstable below 720° C) is not converted into the soft perlite, which is formed at temperatures above ± 500° C, but into the hard martensite formed below 150° C. A closer investigation of the transformation has shown, however, that when it takes place in the temperature range from 200 to 400° C a structure with remarkable mechanical properties is formed (bainite): its hardness is indeed less than that of martensite, but its toughness is considerably greater. The process used to obtain this transformation product is called austempering. Several experiments are described which where carried out with this method of hardening on a special spring steel and which produced good results. For other applications also, for instance in the caps of the "Philishave" electric shaving apparatus, for which the material, in addition to a high resistance against wear, must possess a certain toughness, austempering produces better results than the ordinary method of hardening.

The hardening of steel

In the ordinary process of hardening steel which has been used for centuries, the object to be tempered is first heated to a high temperature, for instance 800° C, whereupon the iron is completely transformed into γ-iron in which the carbon present is dissolved. In this way the so-called austenite structure is formed. Below a temperature of 720° C γ-iron is not stable, upon cooling therefore transformation takes place into α-iron in which carbon is only slightly soluble. The manner of separation of the carbon and thus the structure obtained depends, not only on the content of carbon and other components, but also very much upon the rate of cooling. With slow cooling a more or less coarse lamellar structure (perlite) is in general obtained, which is soft; upon rapid cooling, on the other hand, an irregular structure of fine crystalline needles occurs (martensite), which gives the material the desired great hardness. *Fig. 1* shows photographs of etched surfaces of the various structures mentioned.

A fairly recent investigation by Davenport and Bain [1]) has made it possible to define more precisely the concepts "slow" and "rapid" cooling. These authors immersed test rods of unalloyed steel with 0.78 per cent of carbon [2]), after heating above 720° C, in different baths of molten salts or alloys whose temperatures lay between 0 and 720° C. Each rod was thus quenched to a given temperature below the transformation point and then kept at that temperature. The transformation of the austenite is then found to proceed imperceptibly slowly at first, probably because a sufficient number of nuclei of the new crystal structure must first be formed, and only after a definite time t does the conversion proceed to an appreciable extent. After a time t_2 the conversion is completed. The important point in this process is, that not

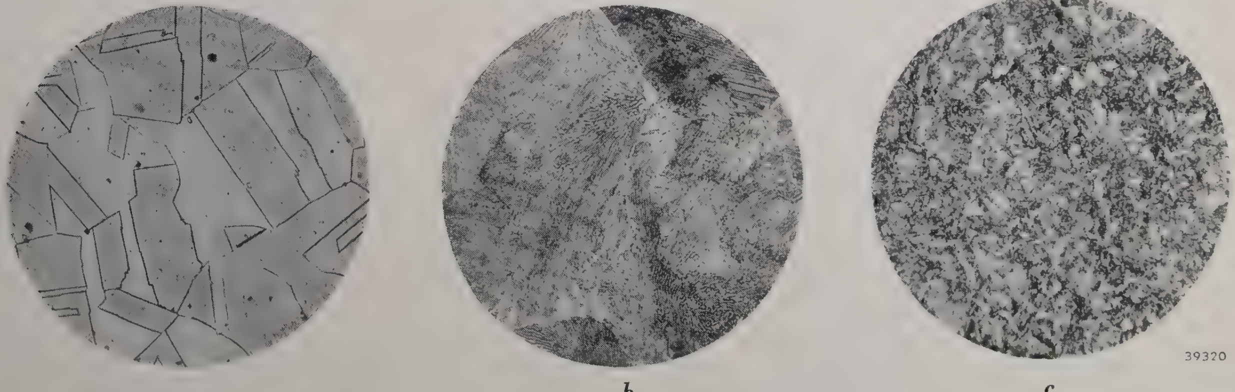


Fig. 1. Etched surfaces of austenite (*a*), perlite (*b*) and martensite (*c*). The photograph (*a*) is of a special kind of steel whose austenite structure decomposes only after a long time even at room temperature. The successive laminae in the perlite structure (*b*) consist alternately of ferrite (pure α-iron) and cementite (Fe_3C).

---

[1]) Trans. Amer. Inst. Mining Metallurg. Eng. (Iron & Steel), 1930, p. 117.

[2]) This carbon content is chosen because at this content there is aneutectic point of the iron-carbon system; at other contents of carbon, upon cooling the steel, a separation of iron or carbon already takes place above the transformation point of the austenite.

only the structure finally formed (and thus the hardness), but also the times t_1 and t_2 depend very closely upon the temperature T of the salt bath, *i.e.* on the temperature at which the transformation takes place.

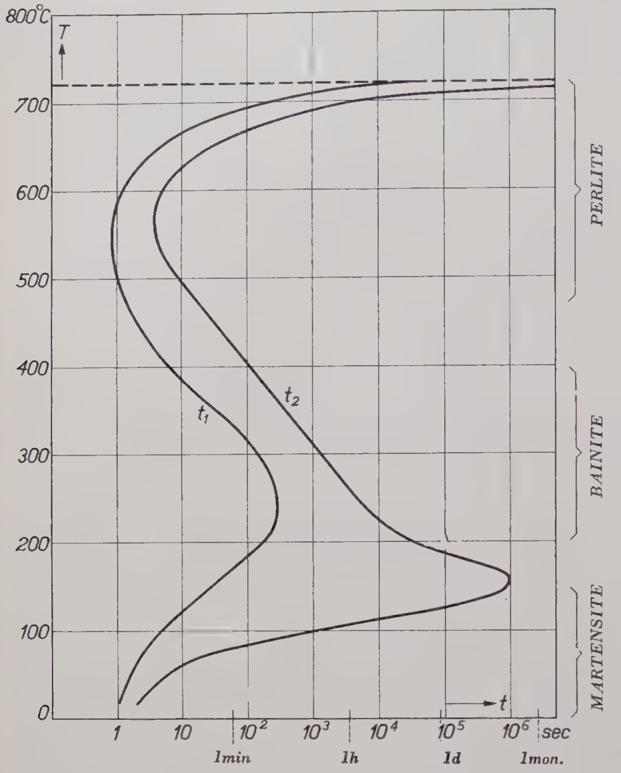


Fig. 2. S-shaped region in which the transformation takes place of the austenite unstable below 720° C. The curves were recorded by the laboratory of the U.S. Steel Corp. (Metal Progress, 1939, p. 374) for an unalloyed steel with 0.78% of carbon. For each temperature T the time t_1 of the beginning and t_2 of the end of the transformation is plotted. On the right the transformation products obtained at the different temperatures are indicated: at high temperatures perlitic structures are formed, below 150° C martensite.

In *fig. 2* the results of this investigation are summarized. Although from the nature of the case the limits and t_2 are not sharp, an S-shaped transformation region is clearly indicated. At temperatures only slightly lower than the transition point, for instance 700° C, the transformation begins only several minutes after quenching, and is complete after about one hour. At temperatures between 500 and 600° C the transformation begins almost immediately and is finished in several seconds. At still lower temperatures, for example from 200 to 300° C, the beginning of the transformation is again very much retarded (5 to 10 minutes) and the transformation occupies several hours. Finally at temperatures in the neighbourhood of room temperature the beginning and end of the transformation again occur very quickly after quenching (several seconds to 1 minute).

In fig. 2 the nature of the transformation product obtained is also indicated. At temperatures only slightly below the transition point, coarse lamellar perlite is obtained; as the transformation temperature becomes lower the perlite becomes finer in structure, until at 500° C the lamellar structure is hardly recognizable any longer. Below about 150° C the austenite is transformed into martensite.

If after having been made austenite by heating, the steel is slowly cooled (normalized), the temperature of the object will vary for example according to line *1* in *fig. 3.* The transformation of the austenite will begin at the temperature T_a and end at the temperature T_b so that the result will be a structure which is a mixture of the soft perlitic structures which correspond according to fig. 2 to the temperature range between T_a and T_b. Upon cooling the heated steel very quickly on the other hand, the temperature of the object varies for instance according to line *2*, the transformation of the austenite now takes place at the low temperatures between T_c and T_d, and a hard structure of martensite is obtained. Therefore in order to harden the steel it must obviously be cooled so quickly that the temperature region between 500 and 600° C is traversed before the transformation can start.

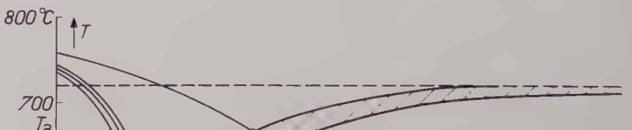


Fig. 3. S-shaped transformation region for a certain kind of steel and schematic variation of temperature upon the use of different methods of cooling the object. *1*, slow cooling (normalizing); the transformation of the austenite takes place at temperatures between T_a and T_b, perlite is formed. *2*, rapid cooling (ordinary tempering); transformation at temperatures between T_c and T_d, martensite is formed. *3*, stepped hardening. *4*, austempering.

The position of the S-shaped transformation region is not the same for all kinds of steel, and the rates of cooling required for hardening differ accordingly. There are kinds of steel for which the dangerous curve of the S is shifted so far to the right that ordinary cooling in air is already "rapid" enough to produce a hard product; in the case of other kinds of steel quenching in oil instead of water is sufficient (due to the higher boiling point and the poorer heat conduction and convection of oil, cooling in oil is much less sudden). In the case of large objects this possibility of a more gradual cooling is very welcome. Upon quenching in water it is only the outer layers of the material which can immediately be cooled to temperatures below 100° C, while the cooling of the inner parts always experiences a certain retardation. This may result in the fact that the inner material does not become hard enough, and due to the uneven cooling and hardening all kinds of stresses and deformations may also occur in the object, expecially since the transformation from austenite into martensite is accompanied by a volume change (expansion of about 1 per cent). Ordinarily the stresses occurring in steel hardened in water are reduced by heating the object for some time after hardening to a temperature of from 150° to 300° C (tempering), whereupon, however, the hardness is somewhat diminished.

Another possibility of combatting strains is the so-called stepped hardening, which has been employed for several decades. In this process the hot object is quenched in a salt bath of for instance 200° C, left in it for several minutes and then slowly cooled further in air. From fig. 3 in which stepped hardening is represented by line *3* it may clearly be seen that also in this case a martensite structure is obtained here when the object is not left too long in the salt bath. Since during this interval before the transformation the object assumes a somewhat more uniform temperature, and then cools further only gradually, the stresses are a great deal smaller. In this case thus the annealing precedes as it were the actual hardening.

Austempering

The American investigations mentioned above have not only given a deeper insight into the different hardening processes, but have also led to the development of a new method of hardening, "austempering". Upon a closer consideration of the different structures which are obtained upon the transformation of austenite at different temperatures, it was found that the transformation product obtained between about 200 and 400° C deserved special attention. Until that time it has been taken simply for very fine lamellar perlite; compared with perlite, however, it was found to possess much greater hardness, namely 400—700 Vickers compared with 200—300 for perlite, while at the same time the toughness of the material was found to be considerably greater than martensite which is tempered to the same hardness. These properties justified the expectation that in some cases where the relatively low hardness is sufficient the martensite could be better replaced by the transformation product in question, which has been given the name "bainite".

Indeed the bainite structure has been put into practical use in America during the last few years. The method of obtaining it is immediately clear after the foregoing: the hot steel is quenched in a salt bath of a temperature of, say 300° C, as is done in stepped hardening. But in this case the steel is left in the bath until the austenite is completely transformed, see line *4* fig. 3. The further cooling may take place rapidly or slowly, the properties of the material show no further appreciable change.

In the Philips factories several experiments have been carried out recently to investigate the practical applicability of this method of tempering with different kinds of steel. In the first place a steel was investigated which is used for making springs, and whose composition is given in *table I*. For this use the toughness of the material is especially important.

Table I

Composition of spring steel for which austempering was tried.

carbon	0.6—0.7%
silicon	0.35%
phosphorus *)	<0.03%
sulphur *)	<0.03%
manganese	0.60%
iron	remainder

*) phosphorus and sulphur together <0.05%.

In order to apply the method of austempering to this material it was first necessary to determine the times necessary for the transformation of austenite at different temperatures, *i.e.* the boundaries of part of the S-shaped transformation region lying between 200 and 400° C had to be found as was done by Davenport and Bain [1]). For this purpose a number of strips of 70 × 8 × 0.8 mm of the steel to be investigated were heated (this was done in an atmosphere of nitrogen in order to limit decarburization and oxidation), and then all were immersed to gether in the salt bath of the temperature T. After different times t a single strip was taken out of the bath and quenched in water. By means of a hardness test the degree could then be determined to which the austenite had been transformed. As long as the times in the salt bath are so short that the conversion of the austenite has not yet begun and the strip in question therefore still consists entirely of austenite, the latter is entirely transformed into martensite upon being quenched in water, and thus attains practically the greatest possible hardness (about 840 Vickers). If a lower hardness is measured it means that in addition to the martensite there is also a softer component present (in this case bainite), and that therefore the transformation of the austenite had already begun in the salt bath. The lower the hardness the farther the conversion had proceeded, until, when the hardness no longer decreases, it may be assumed that the transformation was complete and that the product obtained contains no martensite but only bainite. In *fig. 4* the results of these experiments for a series of temperatures T of the salt bath are represented graphically by plotting the measured hardness as a function of the time t of remaining in the salt bath. From each curve the times t_1 of the beginning and t_2 of the end of the conversion can easily be read off.

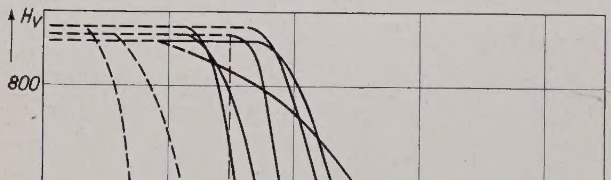


Fig. 4. Hardness measurements of strips of spring steel for determining the S-shaped transformation region. The strips which were quenched in a bath of the temperature T, where each, after a certain time i quenched further in water, and the hardness H_V in Vickers units was then measured. H_V is plotted as a function of t with the temperature T as a parameter. From each curve the time t_1 of the beginning and the time t_2 of the conclusion of the transformation can be read off.

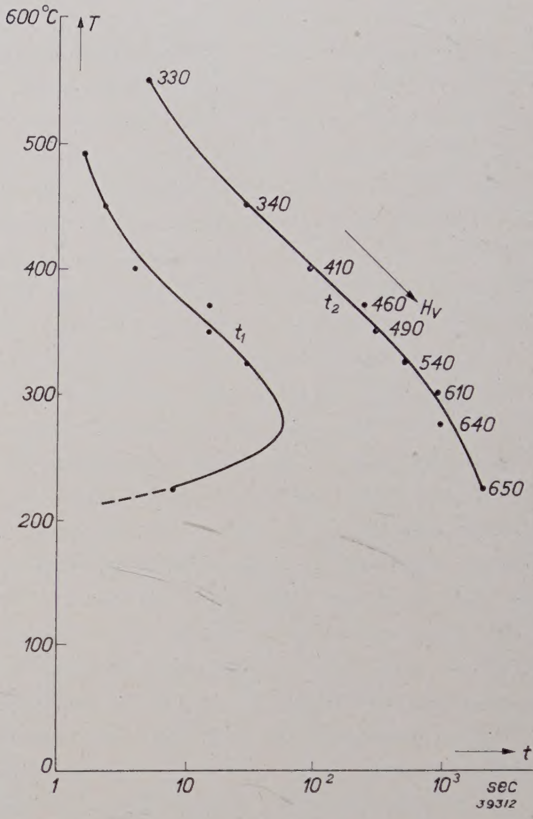
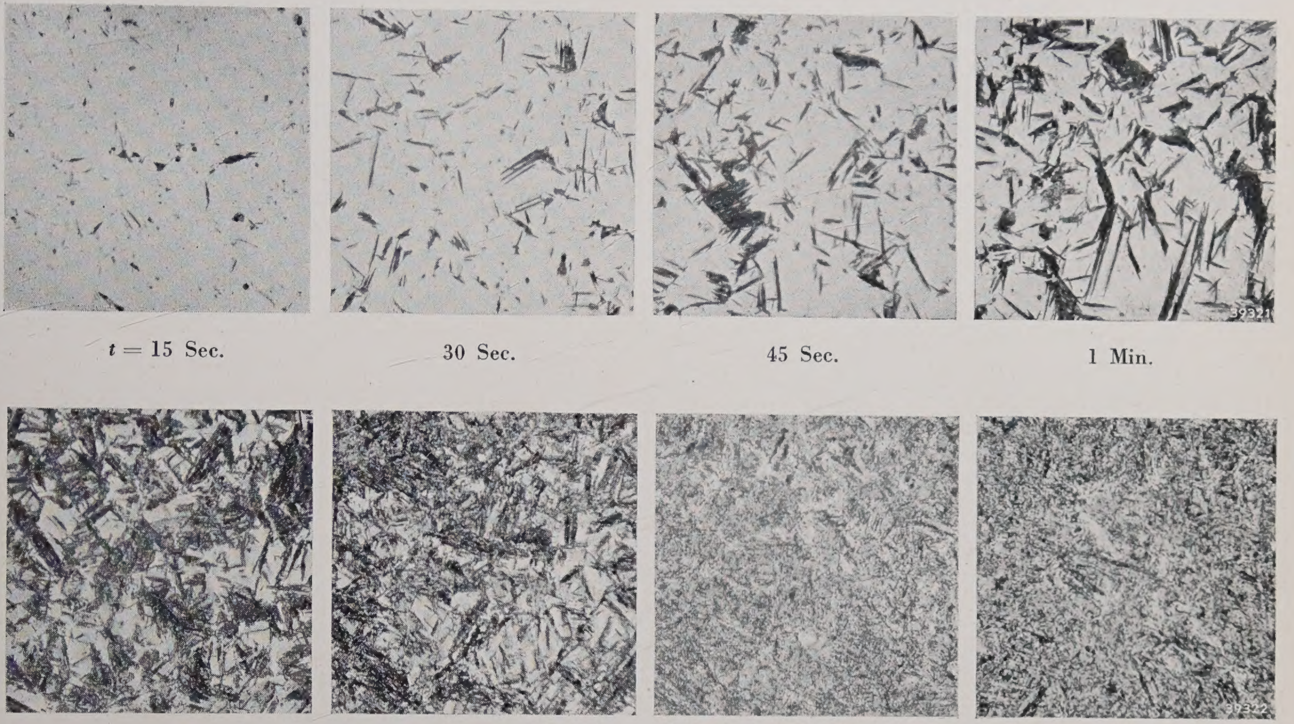


Fig. 5. Transformation region of spring steel. The times t_1 and t_2 from fig. 4 are plotted against the temperature T, while the final hardness obtained H_V (upon complete transformation into bainite) is also indicated.

In *fig. 5* the times t_1 and t_2 deduced from fig. 4 are plotted as functions of the temperature T. In this way the part of the S-region for the spring steel in question, which is of importance in austempering is determined. At the same time the hardness which was finally obtained at each temperature is indicated in the figure.

The course of the transformation may also be followed by microscopic examination of the structure of the strips quenched in water, instead of by hardness masurements. *Fig. 6* shows photographs of the structure of eight strips taken out of the salt bath after different times, at a temperature of the salt bath of 350° C. The light-coloured component in these photomicrographs is martensite, the darker component is the more rapidly etched bainite.

In order to ascertain how the austempered spring steel will behave in practice the toughness and the resiliency was determined of a number of strips which were austempered at different temperatures and thus possessed different hardnesses. The resiliency, which was determined with Tarnogrocki's apparatus by bending the test strip 180° around a mandrel of 20 mm, and measuring the angle through which the strip springs back on removal of the bending force, directly expresses the "springy" qualities of the material. It is closely correlated with the height of the limit of elasticity and thus with the hardness. The toughness, which was determined by measuring the angle through which a test piece can be bent around a mandrel of 1 mm without breaking, provides a measure for the work of deformation which the spring can take up. In *table II* the results of the measurements are summarized. At the same time for the sake of comparison in *table III* the hardness, resilience and toughness are given for a series of strips of the same spring steel which were hardened in oil in the ordinary way and then tempered for 15 minutes at different temperatures.


t = 15 Sec. 30 Sec. 45 Sec. 1 Min.

$t = 1^1/_2$ Min. 2 Min. 3 Min. 4 Min.

Fig. 6. Photographs of etched surfaces of strips of spring steel which were quenched in a bath with a temperature of 350° C and remained in it for different times t, before being further cooled in water. The light-coloured component is martensite, the darker is the more rapidly etched bainite.

It is found from a consideration of the two tables that the austempered material has the greater toughness throughout with a given resiliency (or hardness). The conclusions of the American investigators about the favourable effect of austempering are thus confirmed for the steel here examined. In *fig. 7*, in which the toughness is plotted against the resiliency, the superiority of austempered steel is very obvious.

Table II

Mechanical properties of spring steel which has been austempered at different temperatures T.

T in °C	t in min.	H_V	Resiliency in °	Angle of bending in °
225	120	657	88	47
250	90	647	90	44
275	60	632	86	50
300	40	572	76	55
325	20	505	70	81
350	10	473	65	89
375	$7^1/_2$	428	59	105
400	5	399	54	about 160

Not only for springs, but also for other applications, the combination of qualities of austempered material may be useful, for example in cases where at one and the same time a given minimum toughness and the highest possible resistance to wear are required [3]). An example of this is found in the caps

---

[3]) It must be kept in mind that it can only be the case where very great hardnesses are not required. Tool steel, for example, which must also be tough, can certainly not be austempered, since hardnesses of more than 800 Vickers are required.

Table III

Mechanical properties of spring steel normally hardined in oil, after tempering at different temperatures.

Tempering temperature in °C	H_V	Resiliency in °	Angle of bending in °
275	639	89	27
300	599	85	86
325	556	82	39
350	546	78	43
375	512	75	49
400	485	71	52
425	450	68	54
450	428	64	58
475	401	60	67
500	366	56	73

of the "Philishave" electric shaving apparatus 4). These caps, perforated plates less than 0.1 mm thick, must be extremely resistant to wear since the cutters pass over them as they rotate to cut off the hairs. At the same time, however, the material of the cap may not be too brittle, since otherwise upon falling or being struck there would be danger of its breaking. Experiments actually showed that in the case of the steel of which these caps are made, the simultaneous requirements of toughness and hardness could be better satisfied by austempering.

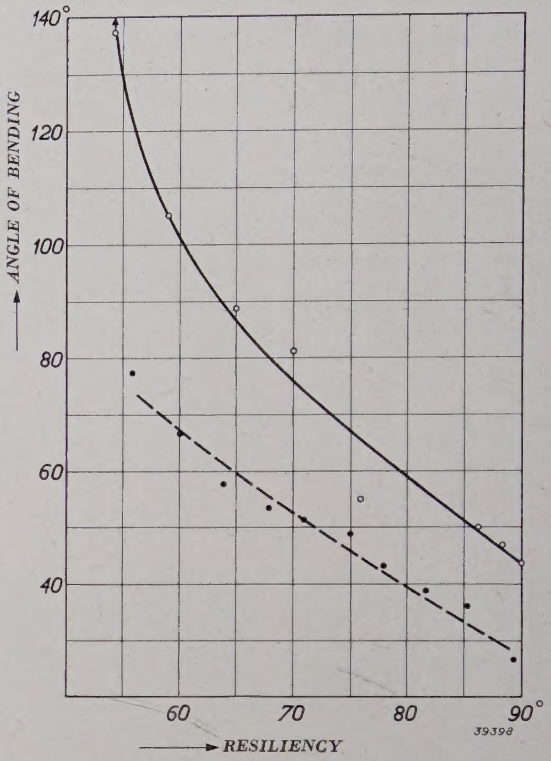


Fig. 7. For a number of normally hardened and tempered strips (brokenline curve) respectively austempering strips (drawn line) the measured toughness (angle measured in bending test) is plotted against the resiliency (angle measured with Tarnogrocki's apparatus).

---

4) Philips techn. Rev. 4, 350, 1939.

RECTIFICATION

In the article of J. F. Schouten, Non-linear Distortion of Sound Film with Oblique Light Slit, Philips techn. Rev. 6, 110, 1941, it is has been presumed that untill the appearance of the cited article an exact mathematical solution of this problem was still outstanding. Dr. A. Narath kindly directs our attention to the fact that already in 1936, be it on an entirely different argumentation, he published an exact solution of this problem in Kinotechnik, 18, 177—180, 1936.